On Curious Family of 3-Tuples

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Contents

Abstract

This book has three sections I, II and III. Section I deals with the study of formulation of special family of 3-tuples (a,b,c) such that the product of any two elements of the set subtracted with their sum is a perfect square. Section II concerns with the process of obtaining family of 3-tuples (a,b,c) such that the product of any two elements of the set added with their sum is a perfect square. Section III illustrates the construction of family of 3- tuples such that the sum of any two members of the set is a perfect square.

Keywords: diophantine 3-tuples, negative pellian equation, integer solutions

Notations

- $SO_n = n(2n^2 - 1) =$ Stella Octangula number of rank n
- $CP_n^3 = \frac{n(n^2+1)}{2} =$ Centered triangular Pyramidal number of rank n
- $CS_n^4 = \frac{n(2n^2+1)}{3} =$ Centered square Pyramidal number of rank n
- $P_n^5 = \frac{n^2(n+1)^3}{2} =$ Pentagonal Pyramidal number of rank n
- $P_n^3 = \frac{n(n+1)(n+2)}{6} =$ Triangular Pyramidal number of rank n
- $CP_{9,n} = \frac{n(3n^2-1)}{2} =$ Centered nonagonal pyramidal number of rank n
- $CP_{24,n} = \frac{24n^3 - 18n}{6} =$ Centered icositetragonal pyramidal number of rank n
- $t_{m,n} = n\left(1 + \frac{(n-1)(m-2)}{2}\right) =$ Polygonal number of rank n with size m
- $PR_n = n(n+1) =$ Pronic number of rank n
- $S_n = 6n(n-1)+1 =$ Star number of rank n
- $GNO_n = 2n - 1 =$ Gnomonic number of rank n

Introduction

The problem of constructing the sets with property that product of any two of its distinct elements is one less than a square has a very long history and such sets have been studied by Diophantus. A set of m distinct positive integers $\{a_1, a_2, a_3, ...,a_m\}$ is said to have the property $D(n), n \in Z - \{0\}$ if $a_i a_j + n$ is a perfect square for all $1 \le i < j \le m$ or $1 \le j < i \le m$ and such a set is called a Diophantine m-tuple with property $D(n)$. In this context, one may refer [1-4].

A set of m distinct positive integers $(a_1, a_2, ..., a_m)$ is said to be Dio m-tuple with property $D(n)$ if $a_i a_j + (a_i + a_j) + n$ or $a_i a_j - (a_i + a_j) + n$ is a perfect square for all $1 \le i < j \le m$ or $1 \le j < i \le m$. In particular, one may refer [5-9] for problem on special dio-3-tuples. Also, one may refer [10-16] for other characterizations of number patterns.

In this book, special families of 3- tuples have been considered. This book has three sections I, II and III. Section I deals with the study of formulation of special family of 3-tuples (a, b, c) such that the product of any two elements of the set subtracted with their sum is a perfect square. Section II deals with the study of formulation of special family of 3-tuples (a, b, c) such that the product of any two elements of the set added with their sum is a perfect square. Section III concerns with the study of formulating 3-tuples consisting of polygonal and pyramidal numbers such that, in each three tuple, the sum of any two members is a perfect square.

Section - I

Triples (a_1, a_2, a_3) such that $a_i a_j - (a_i + a_j) = a\ perfect\ square\ , 1 \le i < j \le 3$

Sequence: 1

Let $a = 2$, $c_0 = k^2 + 2$

It is observed that

$$ac_0 - (a + c_0) = k^2$$

Let c_1 be any integer such that

$$(a-1)c_1 - a = \alpha^2 \quad (1)$$

$$(c_0 - 1)c_1 - c_0 = \beta^2 \quad (2)$$

Eliminating c_1 between (1) and (2), we have

$$(c_0 - 1)\alpha^2 - (a-1)\beta^2 = a - c_0 \quad (3)$$

Introducing the linear transformations

$$\alpha = X + (a-1)T \ , \ \beta = X + (c_0 - 1)T \quad (4)$$

In (3) and simplifying we get

$$X^2 = (a-1)(c_0 - 1)T^2 - 1$$

Which is satisfied by $T = 1$, $X = k$

In view of (4) and (1), it is seen that

$$c_1 = k^2 + 2k + 3$$

Let c_2 be any integer such that

$$(a-1)c_2 - a = \alpha^2 \quad (5)$$

$$(c_1 - 1)c_2 - c_1 = \beta^2 \quad (6)$$

Eliminating c_2 between (5) and (6), we have

$$(c_1-1)\alpha^2-(a-1)\beta^2=a-c_1 \tag{7}$$

Introducing the linear transformations

$$\alpha=X+(a-1)T\ ,\ \beta=X+(c_1-1)T \tag{8}$$

In (7) and simplifying we get

$$X^2=(a-1)(c_1-1)T^2-1$$

Which is satisfied by $T=1\ ,\ X=k+1$

In view of (8) and (5), it is seen that

$$c_2=k^2+4k+6$$

Let c_3 be any integer such that

$$(a-1)\,c_3-a=\alpha^2 \tag{9}$$

$$(c_2-1)\,c_3-c_2=\beta^2 \tag{10}$$

Eliminating c_3 between (9) and (10), we have

$$(c_2-1)\alpha^2-(a-1)\beta^2=a-c_2 \tag{11}$$

Introducing the linear transformations

$$\alpha=X+(a-1)T\ ,\ \beta=X+(c_2-1)T \tag{12}$$

In (11) and simplifying we get

$$X^2=(a-1)(c_2-1)T^2-1$$

Which is satisfied by $T=1\ ,\ X=k+2$

In view of (12) and (9), it is seen that

$$c_3=k^2+6k+11$$

The repetition of the above process leads to the generation of sequence of 3-tuples whose general form is given by $(a\,,c_{s-1}\,,c_s\,)$ where

$$c_{s-1} = k^2 + 2(s-1)k + s^2 - 2s + 3 \ , \ s = 1,2,3,...$$

A few numerical examples are presented in Table: 1 below:

Table 1: Numerical Examples

k	(a, c_0, c_1)	(a, c_1, c_2)	(a, c_2, c_3)
1.	(2 , 6 , 11)	(2 , 11 , 18)	(2 , 18 , 27)
2.	(2 , 11 , 18)	(2 , 18 , 27)	(2 , 27 , 38)
3.	(2 , 18 , 27)	(2 , 27 , 38)	(2 , 38 , 51)
4.	(2 , 27 , 38)	(2 , 38 , 51)	(2 , 51 , 66)

Sequence: 2

Let $a = 3$, $c_0 = 2k^2 - 2k + 2$

It is observed that

$$ac_0 - (a + c_0) = (2k-1)^2$$

Let c_1 be any integer such that

$$(a-1)c_1 - a = \alpha^2 \tag{13}$$

$$(c_0 - 1)c_1 - c_0 = \beta^2 \tag{14}$$

Eliminating c_1 between (13) and (14), we have

$$(c_0 - 1)\alpha^2 - (a-1)\beta^2 = a - c_0 \tag{15}$$

Introducing the linear transformations

$$\alpha = X + (a-1)T \ , \ \beta = X + (c_0 - 1)T \tag{16}$$

In (15) and simplifying we get

$$X^2 = (a-1)(c_0 - 1)T^2 - 1$$

Which is satisfied by $T = 1$, $X = 2k - 1$

In view of (16) and (13), it is seen that

$$c_1 = 2k^2 + 2k + 2$$

Let c_2 be any integer such that

$$(a-1)c_2 - a = \alpha^2 \quad (17)$$

$$(c_1-1)c_2 - c_1 = \beta^2 \quad (18)$$

Eliminating c_2 between (17) and (18), we have

$$(c_1-1)\alpha^2 - (a-1)\beta^2 = a - c_1 \quad (19)$$

Introducing the linear transformations

$$\alpha = X + (a-1)T \ , \ \beta = X + (c_1-1)T \quad (20)$$

In (19) and simplifying we get

$$X^2 = (a-1)(c_1-1)T^2 - 1$$

Which is satisfied by $T = 1$, $X = 2k+1$

In view of (20) and (17), it is seen that

$$c_2 = 2k^2 + 6k + 6$$

Let c_3 be any integer such that

$$(a-1)\,c_3 - a = \alpha^2 \quad (21)$$

$$(c_2-1)\,c_3 - c_2 = \beta^2 \quad (22)$$

Eliminating c_3 between (21) and (22), we have

$$(c_2-1)\alpha^2 - (a-1)\beta^2 = a - c_2 \quad (23)$$

Introducing the linear transformations

$$\alpha = X + (a-1)T \ , \ \beta = X + (c_2-1)T \quad (24)$$

In (23) and simplifying we get

$$X^2 = (a-1)(c_2-1)T^2 - 1$$

Which is satisfied by $T = 1$, $X = 2k+3$

In view of (24) and (21), it is seen that

$$c_3 = 2k^2 + 10k + 14$$

The repetition of the above process leads to the generation of sequence of 3-tuples whose general form is given by (a, c_{s-1}, c_s) where

$$c_{s-1} = 2k^2 + (4s-6)k + 2s^2 - 6s + 6 \ , \ s = 1,2,3,\ldots$$

A few numerical examples are presented in Table: 2 below:

Table 2: Numerical Examples

k	(a, c_0, c_1)	(a, c_1, c_2)	(a, c_2, c_3)
1.	(3 , 6 , 14)	(3 , 14 , 26)	(3 , 26 , 42)
2.	(3 , 14 , 26)	(3 , 26 , 42)	(3 , 42 , 62)
3.	(3 , 26 , 42)	(3 , 42 , 62)	(3 , 62 , 86)
4.	(3 , 42 , 62)	(3 , 62 , 86)	(3 , 86 , 114)

Sequence: 3

Let $a = 6 \ , \ c_0 = 5k^2 - 4k + 2$

It is observed that

$$ac_0 - (a + c_0) = (5k-2)^2$$

Let c_1 be any integer such that

$$(a-1)c_1 - a = \alpha^2 \tag{25}$$

$$(c_0 - 1)c_1 - c_0 = \beta^2 \tag{26}$$

Eliminating c_1 between (25) and (26), we have

$$(c_0 - 1)\alpha^2 - (a-1)\beta^2 = a - c_0 \tag{27}$$

Introducing the linear transformations

$$\alpha = X + (a-1)T \ , \ \beta = X + (c_0 - 1)T \tag{28}$$

In (27) and simplifying we get

$$X^2 = (a-1)(c_0-1)T^2 - 1$$

Which is satisfied by $T = 1 \ , \ X = 5k - 2$

In view of (28) and (25), it is seen that

$$c_1 = 5k^2 + 6k + 3$$

Let c_2 be any integer such that

$$(a-1)c_2 - a = \alpha^2 \tag{29}$$

$$(c_1-1)c_2 - c_1 = \beta^2 \tag{30}$$

Eliminating c_2 between (29) and (30), we have

$$(c_1-1)\alpha^2 - (a-1)\beta^2 = a - c_1 \tag{31}$$

Introducing the linear transformations

$$\alpha = X + (a-1)T \ , \ \beta = X + (c_1-1)T \tag{32}$$

In (31) and simplifying we get

$$X^2 = (a-1)(c_1-1)T^2 - 1$$

Which is satisfied by $T = 1$, $X = 5k + 3$

In view of (32) and (29), it is seen that

$$c_2 = 5k^2 + 16k + 14$$

Let c_3 be any integer such that

$$(a-1)\,c_3 - a = \alpha^2 \tag{33}$$

$$(c_2-1)\,c_3 - c_2 = \beta^2 \tag{34}$$

Eliminating c_3 between (33) and (34), we have

$$(c_2-1)\alpha^2 - (a-1)\beta^2 = a - c_2 \tag{35}$$

Introducing the linear transformations

$$\alpha = X + (a-1)T \ , \ \beta = X + (c_2-1)T \tag{36}$$

In (35) and simplifying we get

$$X^2 = (a-1)(c_2-1)T^2 - 1$$

Which is satisfied by $T=1$, $X=5k+8$

In view of (36) and (33), it is seen that

$$c_3 = 5k^2 + 26k + 35$$

The repetition of the above process leads to the generation of sequence of 3-tuples whose general form is given by (a, c_{s-1}, c_s) where

$$c_{s-1} = 5k^2 + (10s-14)k + 5s^2 - 14s + 11 \ , \ s = 1,2,3,\ldots$$

A few numerical examples are presented in Table: 3 below:

Table 3: Numerical Examples

k	(a, c_0, c_1)	(a, c_1, c_2)	(a, c_2, c_3)
1.	(6 , 14 , 35)	(6 , 35 , 66)	(6 , 66 , 107)
2.	(6 , 35 , 66)	(6 , 66 , 107)	(6 , 107 , 158)
3.	(6 , 66 , 107)	(6 , 107 , 158)	(6 , 158 , 219)
4.	(6 , 107 , 158)	(6 , 158 , 219)	(6 , 219 , 290)

Sequence: 4

Let $a=6$, $c_0 = 5k^2 - 6k + 3$

It is observed that

$$ac_0 - (a + c_0) = (5k-3)^2$$

Let c_1 be any integer such that

$$(a-1)c_1 - a = \alpha^2 \tag{37}$$

$$(c_0-1)c_1 - c_0 = \beta^2 \tag{38}$$

Eliminating c_1 between (37) and (38), we have

$$(c_0-1)\alpha^2 - (a-1)\beta^2 = a - c_0 \tag{39}$$

Introducing the linear transformations

$$\alpha = X + (a-1)T \ , \ \beta = X + (c_0-1)T \tag{40}$$

In (39) and simplifying we get

$$X^2 = (a-1)(c_0 - 1)T^2 - 1$$

Which is satisfied by $T = 1$, $X = 5k - 3$

In view of (40) and (37), it is seen that

$$c_1 = 5k^2 + 4k + 2$$

Let c_2 be any integer such that

$$(a-1)c_2 - a = \alpha^2 \quad (41)$$

$$(c_1 - 1)c_2 - c_1 = \beta^2 \quad (42)$$

Eliminating c_2 between (41) and (42), we have

$$(c_1 - 1)\alpha^2 - (a-1)\beta^2 = a - c_1 \quad (43)$$

Introducing the linear transformations

$$\alpha = X + (a-1)T \ , \ \beta = X + (c_1 - 1)T \quad (44)$$

In (43) and simplifying we get

$$X^2 = (a-1)(c_1 - 1)T^2 - 1$$

Which is satisfied by $T = 1$, $X = 5k + 2$

In view of (44) and (41), it is seen that

$$c_2 = 5k^2 + 14k + 11$$

Let c_3 be any integer such that

$$(a-1)\,c_3 - a = \alpha^2 \quad (45)$$

$$(c_2 - 1)\,c_3 - c_2 = \beta^2 \quad (46)$$

Eliminating c_3 between (45) and (46), we have

$$(c_2 - 1)\alpha^2 - (a-1)\beta^2 = a - c_2 \quad (47)$$

Introducing the linear transformations

$$\alpha = X + (a-1)T \ , \ \beta = X + (c_2 - 1)T \qquad (48)$$

In (47) and simplifying we get

$$X^2 = (a-1)(c_2-1)T^2 - 1$$

Which is satisfied by $T = 1$, $X = 5k + 7$

In view of (48) and (45), it is seen that

$$c_3 = 5k^2 + 24k + 30$$

The repetition of the above process leads to the generation of sequence of 3-tuples whose general form is given by (a, c_{s-1}, c_s) where

$$c_{s-1} = 5k^2 + (10s - 16)k + 5s^2 - 16s + 14 \ , \ s = 1,2,3,\ldots$$

A few numerical examples are presented in Table: 4 below:

Table 4: Numerical Examples

k	(a, c_0, c_1)	(a, c_1, c_2)	(a, c_2, c_3)
1.	(6 , 11 , 30)	(6 , 30 , 59)	(6 , 59 , 98)
2.	(6 , 30 , 59)	(6 , 59 , 98)	(6 , 98 , 147)
3.	(6 , 59 , 98)	(6 , 98 , 147)	(6 , 147 , 206)
4.	(6 , 98 , 147)	(6 , 147 , 206)	(6 , 206 , 275)

Sequence: 5

Let $a = 11$, $c_0 = 10k^2 - 6k + 2$

It is observed that

$$ac_0 - (a + c_0) = (10k - 3)^2$$

Let c_1 be any integer such that

$$(a-1)c_1 - a = \alpha^2 \qquad (49)$$

$$(c_0 - 1)c_1 - c_0 = \beta^2 \qquad (50)$$

Eliminating c_1 between (49) and (50), we have

$$(c_0 - 1)\alpha^2 - (a-1)\beta^2 = a - c_0 \tag{51}$$

Introducing the linear transformations

$$\alpha = X + (a-1)T \ , \ \beta = X + (c_0 - 1)T \tag{52}$$

In (51) and simplifying we get

$$X^2 = (a-1)(c_0 - 1)\,T^2 - 1$$

Which is satisfied by $T = 1 \ , \ X = 10k - 3$

In view of (52) and (49), it is seen that

$$c_1 = 10k^2 + 14k + 6$$

Let c_2 be any integer such that

$$(a-1)c_2 - a = \alpha^2 \tag{53}$$

$$(c_1 - 1)c_2 - c_1 = \beta^2 \tag{54}$$

Eliminating c_2 between (53) and (54), we have

$$(c_1 - 1)\alpha^2 - (a-1)\beta^2 = a - c_1 \tag{55}$$

Introducing the linear transformations

$$\alpha = X + (a-1)T \ , \ \beta = X + (c_1 - 1)T \tag{56}$$

In (55) and simplifying we get

$$X^2 = (a-1)(c_1 - 1)T^2 - 1$$

Which is satisfied by $T = 1 \ , \ X = 10k + 7$

In view of (56) and (53), it is seen that

$$c_2 = 10k^2 + 34k + 30$$

Let c_3 be any integer such that

$$(a-1)\, c_3 - a = \alpha^2 \tag{57}$$

$$(c_2 - 1)\, c_3 - c_2 = \beta^2 \tag{58}$$

Eliminating c_3 between (57) and (58), we have

$$(c_2 - 1)\alpha^2 - (a-1)\beta^2 = a - c_2 \tag{59}$$

Introducing the linear transformations

$$\alpha = X + (a-1)T \ , \ \beta = X + (c_2 - 1)T \tag{60}$$

In (59) and simplifying we get

$$X^2 = (a-1)(c_2 - 1)T^2 - 1$$

Which is satisfied by $T = 1 \ , \ X = 10k + 17$

In view of (60) and (57), it is seen that

$$c_3 = 10k^2 + 54k + 74$$

The repetition of the above process leads to the generation of sequence of 3-tuples whose general form is given by (a, c_{s-1}, c_s) where

$$c_{s-1} = 10k^2 + (20s - 26)k + 10s^2 - 26s + 18 \ , \ s = 1, 2, 3, \ldots$$

A few numerical examples are presented in Table: 5 below:

Table 5: Numerical Examples

k	(a, c_0, c_1)	(a, c_1, c_2)	(a, c_2, c_3)
1.	(11 , 30 , 74)	(11 , 74 , 138)	(11 , 138 , 222)
2.	(11 , 74 , 138)	(11 , 138 , 222)	(11 , 222 , 326)
3.	(11 , 138 , 222)	(11 , 222 , 326)	(11 , 326 , 450)
4.	(11 , 222 , 326)	(11 , 326 , 450)	(11 , 450 , 594)

Sequence: 6

Let $a = 11$, $c_0 = 10k^2 - 14k + 6$

It is observed that

$$ac_0 - (a + c_0) = (10k - 7)^2$$

Let c_1 be any integer such that

$$(a-1)c_1 - a = \alpha^2 \tag{61}$$

$$(c_0-1)c_1 - c_0 = \beta^2 \tag{62}$$

Eliminating c_1 between (61) and (62), we have

$$(c_0-1)\alpha^2 - (a-1)\beta^2 = a - c_0 \tag{63}$$

Introducing the linear transformations

$$\alpha = X + (a-1)T \ , \ \beta = X + (c_0-1)T \tag{64}$$

In (63) and simplifying we get

$$X^2 = (a-1)(c_0-1)T^2 - 1$$

Which is satisfied by $T = 1 \ , \ X = 10k - 7$

In view of (64) and (61), it is seen that

$$c_1 = 10k^2 + 6k + 2$$

Let c_2 be any integer such that

$$(a-1)c_2 - a = \alpha^2 \tag{65}$$

$$(c_1-1)c_2 - c_1 = \beta^2 \tag{66}$$

Eliminating c_2 between (65) and (66), we have

$$(c_1-1)\alpha^2 - (a-1)\beta^2 = a - c_1 \tag{67}$$

Introducing the linear transformations

$$\alpha = X + (a-1)T \ , \ \beta = X + (c_1-1)T \tag{68}$$

In (67) and simplifying we get

$$X^2 = (a-1)(c_1-1)T^2 - 1$$

Which is satisfied by $T = 1 \ , \ X = 10k + 3$

In view of (68) and (65), it is seen that

$$c_2 = 10k^2 + 26k + 18$$

Let c_3 be any integer such that

$$(a-1)\,c_3 - a = \alpha^2 \tag{69}$$

$$(c_2-1)\,c_3 - c_2 = \beta^2 \tag{70}$$

Eliminating c_3 between (69) and (70), we have

$$(c_2-1)\alpha^2 - (a-1)\beta^2 = a - c_2 \tag{71}$$

Introducing the linear transformations

$$\alpha = X + (a-1)T \;,\; \beta = X + (c_2-1)T \tag{72}$$

In (71) and simplifying we get

$$X^2 = (a-1)(c_2-1)T^2 - 1$$

Which is satisfied by $T = 1 \;,\; X = 10k + 13$

In view of (72) and (69), it is seen that

$$c_3 = 10k^2 + 46k + 54$$

The repetition of the above process leads to the generation of sequence of 3-tuples whose general form is given by (a, c_{s-1}, c_s) where

$$c_{s-1} = 10k^2 + (20s - 34)k + 10s^2 - 34s + 30 \;,\; s = 1,2,3,\ldots$$

A few numerical examples are presented in Table: 6 below:

Table 6: Numerical Examples

k	(a, c_0, c_1)	(a, c_1, c_2)	(a, c_2, c_3)
1.	(11 , 18 , 54)	(11 , 54 , 110)	(11 , 110 , 186)
2.	(11 , 54 , 110)	(11 , 110 , 186)	(11 , 186 , 282)
3.	(11 , 110 , 186)	(11 , 186 , 282)	(11 , 282 , 398)
4.	(11 , 186 , 282)	(11 , 282 , 398)	(11 , 398 , 534)

Sequence: 7

Let $a = 14$, $c_0 = 13k^2 - 16k + 6$

It is observed that

$$ac_0 - (a + c_0) = (13k - 8)^2$$

Let c_1 be any integer such that

$$(a-1)c_1 - a = \alpha^2 \qquad (73)$$

$$(c_0 - 1)c_1 - c_0 = \beta^2 \qquad (74)$$

Eliminating c_1 between (73) and (74), we have

$$(c_0 - 1)\alpha^2 - (a-1)\beta^2 = a - c_0 \qquad (75)$$

Introducing the linear transformations

$$\alpha = X + (a-1)T \ , \ \beta = X + (c_0 - 1)T \qquad (76)$$

In (75) and simplifying we get

$$X^2 = (a-1)(c_0 - 1)T^2 - 1$$

Which is satisfied by $T = 1$, $X = 13k - 8$

In view of (76) and (73), it is seen that

$$c_1 = 13k^2 + 10k + 3$$

Let c_2 be any integer such that

$$(a-1)c_2 - a = \alpha^2 \qquad (77)$$

$$(c_1 - 1)c_2 - c_1 = \beta^2 \qquad (78)$$

Eliminating c_2 between (77) and (78), we have

$$(c_1 - 1)\alpha^2 - (a-1)\beta^2 = a - c_1 \qquad (79)$$

Introducing the linear transformations

$$\alpha = X + (a-1)T \ , \ \beta = X + (c_1 - 1)T \qquad (80)$$

In (79) and simplifying we get

$$X^2 = (a-1)(c_1-1)T^2 - 1$$

Which is satisfied by $T = 1$, $X = 13k + 5$

In view of (80) and (77), it is seen that

$$c_2 = 13k^2 + 36k + 26$$

Let c_3 be any integer such that

$$(a-1)\, c_3 - a = \alpha^2 \tag{81}$$

$$(c_2-1)\, c_3 - c_2 = \beta^2 \tag{82}$$

Eliminating c_3 between (81) and (82), we have

$$(c_2-1)\alpha^2 - (a-1)\beta^2 = a - c_2 \tag{83}$$

Introducing the linear transformations

$$\alpha = X + (a-1)T \ , \ \beta = X + (c_2-1)T \tag{84}$$

In (83) and simplifying we get

$$X^2 = (a-1)(c_2-1)T^2 - 1$$

Which is satisfied by $T = 1$, $X = 13k + 18$

In view of (84) and (81), it is seen that

$$c_3 = 13k^2 + 62k + 75$$

The repetition of the above process leads to the generation of sequence of 3-tuples whose general form is given by (a, c_{s-1}, c_s) where

$$c_{s-1} = 13k^2 + (26s - 42)k + 13s^2 - 42s + 35 \ , \ s = 1, 2, 3, \ldots$$

A few numerical examples are presented in Table: 7 below:

Table 7: Numerical Examples

k	(a, c_0, c_1)	(a, c_1, c_2)	(a, c_2, c_3)
1.	(14 , 26 , 75)	(14 , 75 , 150)	(14 , 150 , 251)
2.	(14 , 75 , 150)	(14 , 150 , 251)	(14 , 251 , 378)

3.	(14 , 150 , 251)	(14 , 251 , 378)	(14 , 378 , 531)
4.	(14 , 251 , 378)	(14 , 378 , 531)	(14 , 531 , 710)

Sequence: 8

Let $a = 18$, $c_0 = 17k^2 - 8k + 2$

It is observed that

$$ac_0 - (a + c_0) = (17k - 4)^2$$

Let c_1 be any integer such that

$$(a-1)c_1 - a = \alpha^2 \qquad (85)$$

$$(c_0 - 1)c_1 - c_0 = \beta^2 \qquad (86)$$

Eliminating c_1 between (85) and (86), we have

$$(c_0 - 1)\alpha^2 - (a-1)\beta^2 = a - c_0 \qquad (87)$$

Introducing the linear transformations

$$\alpha = X + (a-1)T \ , \ \beta = X + (c_0 - 1)T \qquad (88)$$

In (87) and simplifying we get

$$X^2 = (a-1)(c_0 - 1)T^2 - 1$$

Which is satisfied by $T = 1$, $X = 17k - 4$

In view of (88) and (85), it is seen that

$$c_1 = 17k^2 + 26k + 11$$

Let c_2 be any integer such that

$$(a-1)c_2 - a = \alpha^2 \qquad (89)$$

$$(c_1 - 1)c_2 - c_1 = \beta^2 \qquad (90)$$

Eliminating c_2 between (89) and (90), we have

$$(c_1 - 1)\alpha^2 - (a-1)\beta^2 = a - c_1 \qquad (91)$$

Introducing the linear transformations

$$\alpha = X + (a-1)T \ , \ \beta = X + (c_1 - 1)T \tag{92}$$

In (91) and simplifying we get

$$X^2 = (a-1)(c_1-1)T^2 - 1$$

Which is satisfied by $T = 1$, $X = 17k + 13$

In view of (92) and (89), it is seen that

$$c_2 = 17k^2 + 60k + 54$$

Let c_3 be any integer such that

$$(a-1)\, c_3 - a = \alpha^2 \tag{93}$$

$$(c_2 - 1)\, c_3 - c_2 = \beta^2 \tag{94}$$

Eliminating c_3 between (93) and (94), we have

$$(c_2 - 1)\alpha^2 - (a-1)\beta^2 = a - c_2 \tag{95}$$

Introducing the linear transformations

$$\alpha = X + (a-1)T \ , \ \beta = X + (c_2 - 1)T \tag{96}$$

in (95) and simplifying we get

$$X^2 = (a-1)(c_2-1)T^2 - 1$$

Which is satisfied by $T = 1$, $X = 17k + 30$

In view of (96) and (93), it is seen that

$$c_3 = 17k^2 + 94k + 131$$

The repetition of the above process leads to the generation of sequence of 3-tuples whose general form is given by (a, c_{s-1}, c_s) where

$$c_{s-1} = 17k^2 + (34s - 42)k + 17s^2 - 42s + 27 \ , \ s = 1, 2, 3, \ldots$$

A few numerical examples are presented in Table: 8 below:

Table 8: Numerical Examples

k	(a, c_0, c_1)	(a, c_1, c_2)	(a, c_2, c_3)
1.	(18 , 54 , 131)	(18 , 131 , 242)	(18 , 242 , 387)
2.	(18 , 131 , 242)	(18 , 242 , 387)	(18 , 387 , 566)
3.	(18 , 242 , 387)	(18 , 387 , 566)	(18 , 566 , 779)
4.	(18 , 387 , 566)	(18 , 566 , 779)	(18 , 779 , 1026)

Sequence: 9

Let $a = 18$, $c_0 = 17k^2 - 26k + 11$

It is observed that

$$ac_0 - (a + c_0) = (17k - 13)^2$$

Let c_1 be any integer such that

$$(a-1)c_1 - a = \alpha^2 \quad (97)$$

$$(c_0 - 1)c_1 - c_0 = \beta^2 \quad (98)$$

Eliminating c_1 between (97) and (98), we have

$$(c_0 - 1)\alpha^2 - (a-1)\beta^2 = a - c_0 \quad (99)$$

Introducing the linear transformations

$$\alpha = X + (a-1)T \ , \ \beta = X + (c_0 - 1)T \quad (100)$$

In (99) and simplifying we get

$$X^2 = (a-1)(c_0 - 1)T^2 - 1$$

Which is satisfied by $T = 1$, $X = 17k - 13$

In view of (100) and (97), it is seen that

$$c_1 = 17k^2 + 8k + 2$$

Let c_2 be any integer such that

$$(a-1)c_2 - a = \alpha^2 \quad (101)$$

$$(c_1 - 1)c_2 - c_1 = \beta^2 \quad (102)$$

Eliminating c_2 between (101) and (102), we have

$$(c_1 - 1)\alpha^2 - (a-1)\beta^2 = a - c_1 \qquad (103)$$

Introducing the linear transformations

$$\alpha = X + (a-1)T \ , \ \beta = X + (c_1 - 1)T \qquad (104)$$

In (103) and simplifying we get

$$X^2 = (a-1)(c_1-1)T^2 - 1$$

Which is satisfied by $T = 1$, $X = 17k + 4$

In view of (104) and (101), it is seen that

$$c_2 = 17k^2 + 42k + 27$$

Let c_3 be any integer such that

$$(a-1)\, c_3 - a = \alpha^2 \qquad (105)$$

$$(c_2 - 1)\, c_3 - c_2 = \beta^2 \qquad (106)$$

Eliminating c_3 between (105) and (106), we have

$$(c_2 - 1)\alpha^2 - (a-1)\beta^2 = a - c_2 \qquad (107)$$

Introducing the linear transformations

$$\alpha = X + (a-1)T \ , \ \beta = X + (c_2 - 1)T \qquad (108)$$

In (107) and simplifying we get

$$X^2 = (a-1)(c_2-1)T^2 - 1$$

Which is satisfied by $T = 1$, $X = 17k + 21$

In view of (108) and (105), it is seen that

$$c_3 = 17k^2 + 76k + 86$$

The repetition of the above process leads to the generation of sequence of 3-tuples whose general form is given by (a, c_{s-1}, c_s) where

$$c_{s-1}=17k^2+(34s-60)k+17s^2-60s+54 \ , \ s=1,2,3,...$$

A few numerical examples are presented in Table: 9 below:

Table 9: Numerical Examples

k	(a,c_0,c_1)	(a,c_1,c_2)	(a,c_2,c_3)
1.	(18 , 27 , 86)	(18 , 86 , 179)	(18 , 179 , 306)
2.	(18 , 86 , 179)	(18 , 179 , 306)	(18 , 306 , 467)
3.	(18 , 179 , 306)	(18 , 306 , 467)	(18 , 467 , 662)
4.	(18 , 306 , 467)	(18 , 467 , 662)	(18 , 662 , 891)

Sequence: 10

Let $a=30$, $c_0=\frac{1}{4}\left(29\left(k^2-k\right)^2+48\left(k^2-k\right)+24\right)$

It is observed that

$$ac_0-(a+c_0)=\frac{1}{4}\left(29\left(k^2-k\right)+24\right)^2$$

Let c_1 be any integer such that

$$(a-1)c_1-a=\alpha^2 \tag{109}$$

$$(c_0-1)c_1-c_0=\beta^2 \tag{110}$$

Eliminating c_1 between (109) and (110), we have

$$(c_0-1)\alpha^2-(a-1)\beta^2=a-c_0 \tag{111}$$

Introducing the linear transformations

$$\alpha=X+(a-1)T \ , \ \beta=X+(c_0-1)T \tag{112}$$

In (111) and simplifying we get

$$X^2=(a-1)(c_0-1)T^2-1$$

Which is satisfied by $T=1$, $X=\frac{1}{2}\left(29\left(k^2-k\right)+24\right)$

In view of (112) and (109), it is seen that

$$c_1 = \frac{1}{4}\left(29\left(k^2 - k\right)^2 + 164\left(k^2 - k\right) + 236\right)$$

Let c_2 be any integer such that

$$(a-1)c_2 - a = \alpha^2 \qquad (113)$$

$$(c_1 - 1)c_2 - c_1 = \beta^2 \qquad (114)$$

Eliminating c_2 between (113) and (114), we have

$$(c_1 - 1)\alpha^2 - (a-1)\beta^2 = a - c_1 \qquad (115)$$

Introducing the linear transformations

$$\alpha = X + (a-1)T \ , \ \beta = X + (c_1 - 1)T \qquad (116)$$

In (115) and simplifying we get

$$X^2 = (a-1)(c_1 - 1)T^2 - 1$$

Which is satisfied by $T = 1$, $X = \frac{1}{2}\left(29\left(k^2 - k\right) + 82\right)$

In view of (116) and (113), it is seen that

$$c_2 = \frac{1}{4}\left(29\left(k^2 - k\right)^2 + 280\left(k^2 - k\right) + 680\right)$$

Let c_3 be any integer such that

$$(a-1)\,c_3 - a = \alpha^2 \qquad (117)$$

$$(c_2 - 1)\,c_3 - c_2 = \beta^2 \qquad (118)$$

Eliminating c_3 between (117) and (118), we have

$$(c_2 - 1)\alpha^2 - (a-1)\beta^2 = a - c_2 \qquad (119)$$

Introducing the linear transformations

$$\alpha = X + (a-1)T \ , \ \beta = X + (c_2 - 1)T \qquad (120)$$

In (119) and simplifying we get

$$X^2 = (a-1)(c_2-1)T^2 - 1$$

Which is satisfied by $T = 1$, $X = \frac{1}{2}\left(29\left(k^2-k\right)+140\right)$

In view of (120) and (117), it is seen that

$$c_3 = \frac{1}{4}\left(29\left(k^2-k\right)^2 + 396\left(k^2-k\right)+1356\right)$$

The repetition of the above process leads to the generation of sequence of 3-tuples whose general form is given by $\left(a, c_{s-1}, c_s\right)$ where

$$c_{s-1} = \frac{1}{4}\left(29\left(k^2-k\right)^2 + (116s-68)\left(k^2-k\right)+116s^2-136s+44\right),\ s=1,2,3,\ldots$$

A few numerical examples are presented in Table: 10 below:

Table 10: Numerical Examples

k	(a, c_0, c_1)	(a, c_1, c_2)	(a, c_2, c_3)
1.	(30 , 59 , 170)	(30 , 170 , 339)	(30 , 339 , 566)
2.	(30 , 339 , 566)	(30 , 566 , 851)	(30 , 851 , 1194)
3.	(30 , 1194 , 1595)	(30 , 1595 , 2054)	(30 , 2054 , 2571)
4.	(30 , 3146 , 3779)	(30 , 3779 , 4470)	(30 , 4470 , 5219)

Sequence: 11

Let $a = 30$, $c_0 = \frac{1}{4}\left(29\left(k^2-k\right)^2 + 68\left(k^2-k\right)+44\right)$

It is observed that

$$ac_0 - (a+c_0) = \frac{1}{4}\left(29\left(k^2-k\right)+34\right)^2$$

Let c_1 be any integer such that

$$(a-1)c_1 - a = \alpha^2 \tag{121}$$

$$(c_0-1)c_1 - c_0 = \beta^2 \tag{122}$$

Eliminating c_1 between (121) and (122), we have

$$(c_0 - 1)\alpha^2 - (a-1)\beta^2 = a - c_0 \tag{123}$$

Introducing the linear transformations

$$\alpha = X + (a-1)T \;,\; \beta = X + (c_0 - 1)T \tag{124}$$

In (123) and simplifying we get

$$X^2 = (a-1)(c_0 - 1)\,T^2 - 1$$

Which is satisfied by $T = 1$, $X = \frac{1}{2}\left(29\left(k^2 - k\right) + 34\right)$

In view of (124) and (121), it is seen that

$$c_1 = \frac{1}{4}\left(29\left(k^2 - k\right)^2 + 184\left(k^2 - k\right) + 296\right)$$

Let c_2 be any integer such that

$$(a-1)c_2 - a = \alpha^2 \tag{125}$$

$$(c_1 - 1)c_2 - c_1 = \beta^2 \tag{126}$$

Eliminating c_2 between (125) and (126), we have

$$(c_1 - 1)\alpha^2 - (a-1)\beta^2 = a - c_1 \tag{127}$$

Introducing the linear transformations

$$\alpha = X + (a-1)T \;,\; \beta = X + (c_1 - 1)T \tag{128}$$

In (127) and simplifying we get

$$X^2 = (a-1)(c_1 - 1)T^2 - 1$$

Which is satisfied by $T = 1$, $X = \frac{1}{2}\left(29\left(k^2 - k\right) + 92\right)$

In view of (128) and (125), it is seen that

$$c_2 = \frac{1}{4}\left(29\left(k^2 - k\right)^2 + 300\left(k^2 - k\right) + 780\right)$$

Let c_3 be any integer such that

$$(a-1)\,c_3 - a = \alpha^2 \tag{129}$$

$$(c_2-1)\,c_3 - c_2 = \beta^2 \tag{130}$$

Eliminating c_3 between (129) and (130), we have

$$(c_2-1)\alpha^2 - (a-1)\beta^2 = a - c_2 \tag{131}$$

Introducing the linear transformations

$$\alpha = X + (a-1)T \;,\; \beta = X + (c_2-1)T \tag{132}$$

In (131) and simplifying we get

$$X^2 = (a-1)(c_2-1)T^2 - 1$$

Which is satisfied by $T = 1$, $X = \frac{1}{2}\left(29\left(k^2-k\right)+150\right)$

In view of (132) and (129), it is seen that

$$c_3 = \frac{1}{4}\left(29\left(k^2-k\right)^2 + 416\left(k^2-k\right)+1496\right)$$

The repetition of the above process leads to the generation of sequence of 3-tuples whose general form is given by $\left(a\,, c_{s-1}\,, c_s\right)$ where

$$c_{s-1} = \frac{1}{4}\left(29\left(k^2-k\right)^2 + (116s-48)\left(k^2-k\right)+116s^2-96s+24\right),\; s = 1,2,3,\ldots$$

A few numerical examples are presented in Table: 11 below:

Table: 11 Numerical Examples

k	(a, c_0, c_1)	(a, c_1, c_2)	(a, c_2, c_3)
1.	(30 , 74 , 195)	(30 , 195 , 374)	(30 , 374 , 611)
2.	(30 , 374 , 611)	(30 , 611 , 906)	(30 , 906 , 1259)
3.	(30 , 1259 , 1670)	(30 , 1670 , 2139)	(30 , 2139 , 2666)
4.	(30 , 3251 , 3894)	(30 , 3894 , 4595)	(30 , 4595 , 5354)

Sequence: 12

Let $a = 42$, $c_0 = 41k^2 - 64k + 26$

It is observed that

$$ac_0 - (a + c_0) = (41k - 32)^2$$

Let c_1 be any integer such that

$$(a-1)c_1 - a = \alpha^2 \qquad (133)$$

$$(c_0 - 1)c_1 - c_0 = \beta^2 \qquad (134)$$

Eliminating c_1 between (133) and (134), we have

$$(c_0 - 1)\alpha^2 - (a-1)\beta^2 = a - c_0 \qquad (135)$$

Introducing the linear transformations

$$\alpha = X + (a-1)T \ , \ \beta = X + (c_0 - 1)T \qquad (136)$$

In (135) and simplifying we get

$$X^2 = (a-1)(c_0 - 1)T^2 - 1$$

Which is satisfied by $T = 1$, $X = 41k - 32$

In view of (136) and (133), it is seen that

$$c_1 = 41k^2 + 18k + 3$$

Let c_2 be any integer such that

$$(a-1)c_2 - a = \alpha^2 \qquad (137)$$

$$(c_1 - 1)c_2 - c_1 = \beta^2 \qquad (138)$$

Eliminating c_2 between (137) and (138), we have

$$(c_1 - 1)\alpha^2 - (a-1)\beta^2 = a - c_1 \qquad (139)$$

Introducing the linear transformations

$$\alpha = X + (a-1)T \ , \ \beta = X + (c_1 - 1)T \qquad (140)$$

In (139) and simplifying we get

$$X^2 = (a-1)(c_1 - 1)T^2 - 1$$

Which is satisfied by $T=1$, $X=41k+9$

In view of (140) and (137), it is seen that

$$c_2=41k^2+100k+62$$

Let c_3 be any integer such that

$$(a-1)\,c_3-a=\alpha^2 \tag{141}$$

$$(c_2-1)\,c_3-c_2=\beta^2 \tag{142}$$

Eliminating c_3 between (141) and (142), we have

$$(c_2-1)\alpha^2-(a-1)\beta^2=a-c_2 \tag{143}$$

Introducing the linear transformations

$$\alpha=X+(a-1)T \ , \ \beta=X+(c_2-1)T \tag{144}$$

In (143) and simplifying we get

$$X^2=(a-1)(c_2-1)T^2-1$$

Which is satisfied by $T=1$, $X=41k+50$

In view of (144) and (141), it is seen that

$$c_3=41k^2+182k+203$$

The repetition of the above process leads to the generation of sequence of 3-tuples whose general form is given by $(a\,,c_{s-1}\,,c_s\,)$ where

$$c_{s-1}=41k^2+(82s-146\,)k+41s^2-146s+131 \ , \ s=1,2,3,...$$

A few numerical examples are presented in Table: 12 below:

Table 12: Numerical Examples

k	(a,c_0,c_1)	(a,c_1,c_2)	(a,c_2,c_3)
2	(42 , 62 , 203)	(42 , 203 , 426)	(42 , 426 , 731)
3	(42 , 203 , 426)	(42 , 426 , 731)	(42 , 731 , 1118)
4	(42 , 426 , 731)	(42 , 731 , 1118)	(42 , 1118 , 1587)
5	(42 , 731 , 1118)	(42 , 1118 , 1587)	(42 , 1587 , 2138)

Sequence: 13

Let $a = 54$, $c_0 = 53k^2 - 60k + 18$

It is observed that

$$ac_0 - (a + c_0) = (53k - 30)^2$$

Let c_1 be any integer such that

$$(a-1)c_1 - a = \alpha^2 \tag{145}$$

$$(c_0 - 1)c_1 - c_0 = \beta^2 \tag{146}$$

Eliminating c_1 between (145) and (146), we have

$$(c_0 - 1)\alpha^2 - (a-1)\beta^2 = a - c_0 \tag{147}$$

Introducing the linear transformations

$$\alpha = X + (a-1)T \ , \ \beta = X + (c_0 - 1)T \tag{148}$$

In (147) and simplifying we get

$$X^2 = (a-1)(c_0 - 1)T^2 - 1$$

Which is satisfied by $T = 1$, $X = 53k - 30$

In view of (148) and (145), it is seen that

$$c_1 = 53k^2 + 46k + 11$$

Let c_2 be any integer such that

$$(a-1)c_2 - a = \alpha^2 \tag{149}$$

$$(c_1 - 1)c_2 - c_1 = \beta^2 \tag{150}$$

Eliminating c_2 between (149) and (150), we have

$$(c_1 - 1)\alpha^2 - (a-1)\beta^2 = a - c_1 \tag{151}$$

Introducing the linear transformations

$$\alpha = X + (a-1)T \ , \ \beta = X + (c_1 - 1)T \tag{152}$$

In (151) and simplifying we get

$$X^2=(a-1)(c_1-1)T^2-1$$

Which is satisfied by $T=1$, $X=53k+23$

In view of (152) and (149), it is seen that

$$c_2=53k^2+152k+110$$

Let c_3 be any integer such that

$$(a-1)\,c_3-a=\alpha^2 \tag{153}$$

$$(c_2-1)\,c_3-c_2=\beta^2 \tag{154}$$

Eliminating c_3 between (153) and (154), we have

$$(c_2-1)\alpha^2-(a-1)\beta^2=a-c_2 \tag{155}$$

Introducing the linear transformations

$$\alpha=X+(a-1)T\ ,\ \beta=X+(c_2-1)T \tag{156}$$

In (155) and simplifying we get

$$X^2=(a-1)(c_2-1)T^2-1$$

Which is satisfied by $T=1$, $X=53k+76$

In view of (156) and (153), it is seen that

$$c_3=53k^2+258k+315$$

The repetition of the above process leads to the generation of sequence of 3-tuples whose general form is given by $(a\,,c_{s-1}\,,c_s\,)$ where

$$c_{s-1}=53k^2+(106s-166)k+53s^2-166s+131\ ,\ s=1,2,3,...$$

A few numerical examples are presented in Table: 13 below:

Table 13: Numerical Examples

k	(a,c_0,c_1)	(a,c_1,c_2)	(a,c_2,c_3)
2	(54 , 110 , 315)	(54 , 315 , 626)	(54 , 626 , 1043)
3	(54 , 315 , 626)	(54 , 626 , 1043)	(54 , 1043 , 1566)
4	(54 , 626 , 1043)	(54 , 1043 , 1566)	(54 , 1566 , 2195)

5	(54 , 1043 , 1566)	(54 , 1566 , 2195)	(54 , 2195 , 2930)

Sequence: 14

Let $a = 62$, $c_0 = 61k^2 - 100k + 42$

It is observed that

$$ac_0 - (a + c_0) = (61k - 50)^2$$

Let c_1 be any integer such that

$$(a-1)c_1 - a = \alpha^2 \tag{157}$$

$$(c_0 - 1)c_1 - c_0 = \beta^2 \tag{158}$$

Eliminating c_1 between (157) and (158), we have

$$(c_0 - 1)\alpha^2 - (a-1)\beta^2 = a - c_0 \tag{159}$$

Introducing the linear transformations

$$\alpha = X + (a-1)T \ , \ \beta = X + (c_0 - 1)T \tag{160}$$

in (159) and simplifying we get

$$X^2 = (a-1)(c_0 - 1)T^2 - 1$$

which is satisfied by $T = 1$, $X = 61k - 50$

In view of (160) and (157), it is seen that

$$c_1 = 61k^2 + 22k + 3$$

Let c_2 be any integer such that

$$(a-1)c_2 - a = \alpha^2 \tag{161}$$

$$(c_1 - 1)c_2 - c_1 = \beta^2 \tag{162}$$

Eliminating c_2 between (161) and (162), we have

$$(c_1 - 1)\alpha^2 - (a-1)\beta^2 = a - c_1 \tag{163}$$

Introducing the linear transformations

$$\alpha = X + (a-1)T \ , \ \beta = X + (c_1 - 1)T \tag{164}$$

In (163) and simplifying we get

$$X^2 = (a-1)(c_1-1)T^2 - 1$$

Which is satisfied by $T = 1 \ , \ X = 61k + 11$

In view of (164) and (161), it is seen that

$$c_2 = 61k^2 + 144k + 86$$

Let c_3 be any integer such that

$$(a-1)\,c_3 - a = \alpha^2 \tag{165}$$

$$(c_2-1)\,c_3 - c_2 = \beta^2 \tag{166}$$

Eliminating c_3 between (165) and (166), we have

$$(c_2-1)\alpha^2 - (a-1)\beta^2 = a - c_2 \tag{167}$$

Introducing the linear transformations

$$\alpha = X + (a-1)T \ , \ \beta = X + (c_2 - 1)T \tag{168}$$

In (167) and simplifying we get

$$X^2 = (a-1)(c_2-1)T^2 - 1$$

Which is satisfied by $T = 1 \ , \ X = 61k + 72$

In view of (168) and (165), it is seen that

$$c_3 = 61k^2 + 266k + 291$$

The repetition of the above process leads to the generation of sequence of 3-tuples whose general form is given by (a, c_{s-1}, c_s) where

$$c_{s-1} = 61k^2 + (122s - 222)k + 61s^2 - 222s + 203 \ , \ s = 1,2,3,\ldots$$

A few numerical examples are presented in Table: 14 below:

Table 14: Numerical Examples

k	(a, c_0, c_1)	(a, c_1, c_2)	(a, c_2, c_3)
1.	(62, 3, 86)	(62, 86, 291)	(62, 291, 618)
2.	(62, 86, 291)	(62, 291, 618)	(62, 618, 1067)
3.	(62, 291, 618)	(62, 618, 1067)	(62, 1067, 1638)
4.	(62, 618, 1067)	(62, 1067, 1638)	(62, 1638, 2331)

Sequence: 15

Let $a = k^2 + 2$, $c_0 = (k^2+1)s^2 - 2s(k^2 - k + 1) + k^2 - 2k + 3$

It is observed that

$$ac_0 - (a + c_0) = (k + (s-1)(k^2+1))^2$$

Let c_1 be any integer such that

$$(a-1)c_1 - a = \alpha^2 \qquad (169)$$

$$(c_0 - 1)c_1 - c_0 = \beta^2 \qquad (170)$$

Eliminating c_1 between (169) and (170), we have

$$(c_0 - 1)\alpha^2 - (a-1)\beta^2 = a - c_0 \qquad (171)$$

Introducing the linear transformations

$$\alpha = X + (a-1)T \ , \ \beta = X + (c_0 - 1)T \qquad (172)$$

In (171) and simplifying we get

$$X^2 = (a-1)(c_0-1)\,T^2 - 1$$

Which is satisfied by $T = 1$, $X = k + (s-1)(k^2+1)$

In view of (172) and (169), it is seen that

$$c_1 = s^2(k^2+1) + 2sk + 2$$

Let c_2 be any integer such that

$$(a-1)c_2 - a = \alpha^2 \qquad (173)$$

$$(c_1 - 1)c_2 - c_1 = \beta^2 \qquad (174)$$

Eliminating c_2 between (173) and (174), we have

$$(c_1-1)\alpha^2-(a-1)\beta^2=a-c_1 \tag{175}$$

Introducing the linear transformations

$$\alpha=X+(a-1)T\ ,\ \beta=X+(c_1-1)T \tag{176}$$

in (175) and simplifying we get

$$X^2=(a-1)(c_1-1)T^2-1$$

which is satisfied by $T=1\ ,\ X=k+s(k^2+1)$

In view of (176) and (173), it is seen that

$$c_2=(s+1)^2(k^2+1)+2sk+2k+2$$

Let c_3 be any integer such that

$$(a-1)\,c_3-a=\alpha^2 \tag{177}$$

$$(c_2-1)\,c_3-c_2=\beta^2 \tag{178}$$

Eliminating c_3 between (177) and (178), we have

$$(c_2-1)\alpha^2-(a-1)\beta^2=a-c_2 \tag{179}$$

Introducing the linear transformations

$$\alpha=X+(a-1)T\ ,\ \beta=X+(c_2-1)T \tag{180}$$

In (179) and simplifying we get

$$X^2=(a-1)(c_2-1)T^2-1$$

Which is satisfied by $T=1\ ,\ X=k+(s+1)(k^2+1)$

In view of (180) and (177), it is seen that

$$c_3=(s+2)^2(k^2+1)+2k(s+2)+2$$

The repetition of the above process leads to the generation of sequence of 3-tuples whose general form is given by (a, c_{n-1}, c_n) where $c_{n-1} = (s+n-2)^2 (k^2+1) + 2k(s+n-2) + 2$, $n = 1, 2, 3, \ldots$

Section - II

Triples (a_1, a_2, a_3) such that $a_i a_j + (a_i + a_j) = a \textit{ perfect square}, 1 \le i < j \le 3$

Sequence: 1

Let $a = 2k^2 + 6k + 4, c_0 = 8k^2 + 16k + 9$

It is observed that

$$ac_0 + a + c_0 = (4k^2 + 10k + 7)^2$$

Let c_1 be any integer such that

$$(a+1)c_1 + a = \alpha^2 \tag{1}$$

$$(c_0 + 1)c_1 + c_0 = \beta^2 \tag{2}$$

Eliminating c_1 between (1) and (2), we have

$$(c_0 + 1)\alpha^2 - (a+1)\beta^2 = (a - c_0) \tag{3}$$

Introducing the linear transformations

$$\alpha = X + (a+1)T \quad , \quad \beta = X + (c_0 + 1)T \tag{4}$$

in (3) and simplifying we get

$$X^2 = (a+1)(c_0 + 1)T^2 - 1$$

which is satisfied by $T = 1$, $X = 4k^2 + 10k + 7$

In view of (4) and (1), it is seen that

$$c_1 = 18k^2 + 42k + 28$$

Let c_2 be any integer such that

$$(a+1)c_2 + a = \alpha^2 \tag{5}$$

$$(c_1+1)c_2+c_1=\beta^2 \tag{6}$$

Eliminating c_2 between (5) and (6), we have

$$(c_1+1)\alpha^2-(a+1)\beta^2=(a-c_1) \tag{7}$$

Introducing the linear transformations

$$\alpha=X+(a+1)T \ , \ \beta=X+(c_1+1)T \tag{8}$$

In (7) and simplifying we get

$$X^2=(a+1)(c_1+1)T^2-1$$

Which is satisfied by $T=1 \ , \ X=6k^2+16k+12$

In view of (8) and (5), it is seen that

$$c_2=32k^2+80k+57$$

Let c_3 be any integer such that

$$(a+1)c_3+a=\alpha^2 \tag{9}$$

$$(c_2+1)c_3+c_2=\beta^2 \tag{10}$$

Eliminating c_3 between (9) and (10), we have

$$(c_2+1)\alpha^2-(a+1)\beta^2=(a-c_2) \tag{11}$$

Introducing the linear transformations

$$\alpha=X+(a+1)T \ , \ \beta=X+(c_2+1)T \tag{12}$$

In (11) and simplifying we get

$$X^2=(a+1)(c_2+1)T^2-1$$

Which is satisfied by $T=1 \ , \ X=8k^2+22k+17$

In view of (12) and (9), it is seen that

$$c_3=50k^2+130k+96$$

The repetition of the above process leads to the generation of sequence of 3-tuples whose general form is given by (a, c_{s-1}, c_s) where

$$c_{s-1} = (2s^2+4s+2)k^2 + (6s^2+8s+2)k + (5s^2+4s) \ , \ s = 1, 2, 3, \ldots$$

A few numerical examples are presented in Table 1 below:

Table 1: Numerical Examples

k	(a, c_0, c_1)	(a, c_1, c_2)	(a, c_2, c_3)	(a, c_3, c_4)
2	(24, 73, 184)	(24, 184, 345)	(24, 345,556)	(24, 556,817)
3	(40, 129, 316)	(40, 316, 585)	(40, 585, 936)	(40, 936, 1369)
4	(60, 201,484)	(60, 484,889)	(60,889,1416)	(60,1416,2065)
5	(84,289,688)	(84,688,1257)	(84,1257,1996)	(84,1996,2905)

Sequence: 2

Let $a = 1, c_0 = 2k^2 - 2k$

It is observed that

$$ac_0 + a + c_0 = (2k-1)^2$$

Let c_1 be any integer such that

$$(a+1)c_1 + a = \alpha^2 \tag{13}$$

$$(c_0+1)c_1 + c_0 = \beta^2 \tag{14}$$

Eliminating c_1 between (13) and (14), we have

$$(c_0+1)\alpha^2 - (a+1)\beta^2 = (a - c_0) \tag{15}$$

Introducing the linear transformations

$$\alpha = X + (a+1)T \ , \ \beta = X + (c_0+1)T \tag{16}$$

In (15) and simplifying we get

$$X^2 = (a+1)(c_0+1)T^2 - 1$$

Which is satisfied by $T = 1 \ , \ X = 2k - 1$

In view of (16) and (13), it is seen that

$$c_1 = 2k^2 + 2k$$

Let c_2 be any integer such that

$$(a+1)c_2 + a = \alpha^2 \tag{17}$$

$$(c_1+1)c_2 + c_1 = \beta^2 \tag{18}$$

Eliminating c_2 between (17) and (18), we have

$$(c_1+1)\alpha^2 - (a+1)\beta^2 = (a - c_1) \tag{19}$$

Introducing the linear transformations

$$\alpha = X + (a+1)T \ , \ \beta = X + (c_1+1)T \tag{20}$$

In (19) and simplifying we get

$$X^2 = (a+1)(c_1+1)T^2 - 1$$

Which is satisfied by $T = 1$, $X = 2k+1$

In view of (20) and (17), it is seen that

$$c_2 = 2k^2 + 6k + 4$$

Let c_3 be any integer such that

$$(a+1)c_3 + a = \alpha^2 \tag{21}$$

$$(c_2+1)c_3 + c_2 = \beta^2 \tag{22}$$

Eliminating c_3 between (21) and (22), we have

$$(c_2+1)\alpha^2 - (a+1)\beta^2 = (a - c_2) \tag{23}$$

Introducing the linear transformations

$$\alpha = X + (a+1)T \ , \ \beta = X + (c_2+1)T \tag{24}$$

In (23) and simplifying we get

$$X^2 = (a+1)(c_2+1)T^2 - 1$$

Which is satisfied by $T = 1$, $X = 2k + 3$

In view of (24) and (21), it is seen that

$$c_3 = 2k^2 + 10k + 12$$

The repetition of the above process leads to the generation of sequence of 3-tuples whose general form is given by (a, c_{s-1}, c_s) where

$$c_{s-1} = 2k^2 + (4s-6)k + (2s^2 - 6s + 4) \ , \ s = 1, 2, 3, \ldots$$

A few numerical examples are presented in Table 2 below:

Table 2: Numerical Examples

k	(a, c_0, c_1)	(a, c_1, c_2)	(a, c_2, c_3)	(a, c_3, c_4)
1.	(1, 4, 12)	(1, 12, 24)	(1, 24, 40)	(1, 40, 60)
2.	(1, 12, 24)	(1, 24, 40)	(1, 40, 60)	(1, 60, 84)
3.	(1, 24,40)	(1, 40, 60)	(1,60, 84)	(1, 84, 112)
4.	(1, 40, 60)	(1, 60, 84)	(1, 84, 112)	(1, 112, 144)

Sequence 3:

Let $a = 1, c_0 = 2k^2 + 2k$

It is observed that

$$ac_0 + a + c_0 = (2k+1)^2$$

Let c_1 be any integer such that

$$(a+1)c_1 + a = \alpha^2 \quad (25)$$

$$(c_0+1)c_1 + c_0 = \beta^2 \quad (26)$$

Eliminating c_1 between (25) and (26), we have

$$(c_0+1)\alpha^2 - (a+1)\beta^2 = (a - c_0) \quad (27)$$

Introducing the linear transformations

$$\alpha = X + (a+1)T \ , \ \beta = X + (c_0+1)T \quad (28)$$

In (27) and simplifying we get

$$X^2 = (a+1)(c_0+1)T^2 - 1$$

Which is satisfied by $T=1$, $X=2k+1$

In view of (28) and (25), it is seen that

$$c_1 = 2k^2 + 6k + 4$$

Let c_2 be any integer such that

$$(a+1)c_2 + a = \alpha^2 \quad (29)$$

$$(c_1+1)c_2 + c_1 = \beta^2 \quad (30)$$

Eliminating c_2 between (29) and (30), we have

$$(c_1+1)\alpha^2 - (a+1)\beta^2 = (a - c_1) \quad (31)$$

Introducing the linear transformations

$$\alpha = X + (a+1)T \; , \; \beta = X + (c_1+1)T \quad (32)$$

In (31) and simplifying we get

$$X^2 = (a+1)(c_1+1)T^2 - 1$$

Which is satisfied by $T=1$, $X=2k+3$

In view of (32) and (29), it is seen that

$$c_2 = 2k^2 + 10k + 12$$

Let c_3 be any integer such that

$$(a+1)c_3 + a = \alpha^2 \quad (33)$$

$$(c_2+1)c_3 + c_2 = \beta^2 \quad (34)$$

Eliminating c_3 between (33) and (34), we have

$$(c_2+1)\alpha^2 - (a+1)\beta^2 = (a - c_2) \quad (35)$$

Introducing the linear transformations

$$\alpha = X + (a+1)T \ , \ \beta = X + (c_2+1)T \tag{36}$$

In (35) and simplifying we get

$$X^2 = (a+1)(c_2+1)T^2 - 1$$

Which is satisfied by $T = 1 \ , \ X = 2k+5$

In view of (36) and (33), it is seen that

$$c_3 = 2k^2 + 14k + 24$$

The repetition of the above process leads to the generation of sequence of 3-tuples whose general form is given by (a, c_{s-1}, c_s) where

$$c_{s-1} = 2k^2 + (4s-2)k + (2s^2 - 2s) \ , \ s = 1, 2, 3, \ldots$$

A few numerical examples are presented in Table 3 below:

Table 3: Numerical Examples

k	(a, c_0, c_1)	(a, c_1, c_2)	(a, c_2, c_3)	(a, c_3, c_4)
1.	(1,12,24)	(1, 24, 40)	(1, 40, 60)	(1, 60, 84)
2.	(1,24,40)	(1, 40, 60)	(1, 60, 84)	(1, 84, 112)
3.	(1,40,60)	(1, 60, 84)	(1, 84, 112)	(1, 112, 144)
4.	(1,60,84)	(1, 84, 112)	(1, 112, 144)	(1, 144, 180)

Sequence: 4

Let $a = 4, c_0 = 5k^2 + 4k$

It is observed that

$$ac_0 + a + c_0 = (5k+2)^2$$

Let c_1 be any integer such that

$$(a+1)c_1 + a = \alpha^2 \tag{37}$$

$$(c_0+1)c_1 + c_0 = \beta^2 \tag{38}$$

Eliminating c_1 between (37) and (38), we have

$$(c_0+1)\alpha^2-(a+1)\beta^2=(a-c_0) \quad (39)$$

Introducing the linear transformations

$$\alpha=X+(a+1)T \ , \ \beta=X+(c_0+1)T \quad (40)$$

In (39) and simplifying we get

$$X^2=(a+1)(c_0+1)T^2-1$$

Which is satisfied by $T=1$, $X=5k+2$

In view of (40) and (37), it is seen that

$$c_1=5k^2+14k+9$$

Let c_2 be any integer such that

$$(a+1)c_2+a=\alpha^2 \quad (41)$$

$$(c_1+1)c_2+c_1=\beta^2 \quad (42)$$

Eliminating c_2 between (41) and (42), we have

$$(c_1+1)\alpha^2-(a+1)\beta^2=(a-c_1) \quad (43)$$

Introducing the linear transformations

$$\alpha=X+(a+1)T \ , \ \beta=X+(c_1+1)T \quad (44)$$

In (43) and simplifying we get

$$X^2=(a+1)(c_1+1)T^2-1$$

Which is satisfied by $T=1$, $X=5k+7$

In view of (44) and (41), it is seen that

$$c_2=5k^2+24k+28$$

Let c_3 be any integer such that

$$(a+1)c_3+a=\alpha^2 \quad (45)$$

$$(c_2+1)c_3+c_2=\beta^2 \quad (46)$$

Eliminating c_3 between (45) and (46), we have

$$(c_2+1)\alpha^2-(a+1)\beta^2=(a-c_2) \quad (47)$$

Introducing the linear transformations

$$\alpha = X+(a+1)T \ , \ \beta = X+(c_2+1)T \quad (48)$$

in (47) and simplifying we get

$$X^2=(a+1)(c_2+1)T^2-1$$

which is satisfied by $T=1 \ , \ X=5k+12$

In view of (48) and (45), it is seen that

$$c_3=5k^2+34k+57$$

The repetition of the above process leads to the generation of sequence of 3-tuples whose general form is given by (a, c_{s-1}, c_s) where

$$c_{s-1}=5k^2+(10s-6)k+(5s^2-6s+1) \ , \ s=1,2,3,...$$

A few numerical examples are presented in Table 4 below:

Table 4: Numerical Examples

k	(a,c_0,c_1)	(a,c_1,c_2)	(a,c_2,c_3)	(a,c_3,c_4)
1.	(4,28,57)	(4, 57, 96)	(4, 96, 145)	(4, 145, 204)
2.	(4,57,96)	(4, 96, 145)	(4, 145, 204)	(4, 204, 273)
3.	(4,96,145)	(4, 145, 204)	(4, 204, 273)	(4, 273, 352)
4.	(4,145,204)	(4, 204, 273)	(4, 273, 352)	(4, 352, 441)

Sequence: 5

Let $a=4, c_0=5k^2-4k$

It is observed that

$$ac_0+a+c_0=(5k-2)^2$$

Let c_1 be any integer such that

$$(a+1)c_1+a=\alpha^2 \quad (49)$$

$$(c_0+1)c_1+c_0=\beta^2 \tag{50}$$

Eliminating c_1 between (49) and (50), we have

$$(c_0+1)\alpha^2-(a+1)\beta^2=(a-c_0) \tag{51}$$

Introducing the linear transformations

$$\alpha=X+(a+1)T \ , \ \beta=X+(c_0+1)T \tag{52}$$

in (51) and simplifying we get

$$X^2=(a+1)(c_0+1)T^2-1$$

which is satisfied by $T=1 \ , \ X=5k-2$

In view of (52) and (49), it is seen that

$$c_1=5k^2+6k+1$$

Let c_2 be any integer such that

$$(a+1)c_2+a=\alpha^2 \tag{53}$$

$$(c_1+1)c_2+c_1=\beta^2 \tag{54}$$

Eliminating c_2 between (53) and (54), we have

$$(c_1+1)\alpha^2-(a+1)\beta^2=(a-c_1) \tag{55}$$

Introducing the linear transformations

$$\alpha=X+(a+1)T \ , \ \beta=X+(c_1+1)T \tag{56}$$

In (55) and simplifying we get

$$X^2=(a+1)(c_1+1)T^2-1$$

Which is satisfied by $T=1 \ , \ X=5k+3$

In view of (56) and (53), it is seen that

$$c_2=5k^2+16k+12$$

Let c_3 be any integer such that

$$(a+1)c_3 + a = \alpha^2 \tag{57}$$

$$(c_2+1)c_3 + c_2 = \beta^2 \tag{58}$$

Eliminating c_3 between (57) and (58), we have

$$(c_2+1)\alpha^2 - (a+1)\beta^2 = (a-c_2) \tag{59}$$

Introducing the linear transformations

$$\alpha = X + (a+1)T \ , \ \beta = X + (c_2+1)T \tag{60}$$

in (59) and simplifying we get

$$X^2 = (a+1)(c_2+1)T^2 - 1$$

which is satisfied by $T = 1 \ , \ X = 5k+8$

In view of (60) and (57), it is seen that

$$c_3 = 5k^2 + 26k + 33$$

The repetition of the above process leads to the generation of sequence of 3-tuples whose general form is given by (a, c_{s-1}, c_s) where

$$c_{s-1} = 5k^2 + (10s-14)k + (5s^2 - 14s + 9) \ , \ s = 1, 2, 3, \ldots$$

A few numerical examples are presented in Table 5 below:

Table 5: Numerical Examples

k	(a, c_0, c_1)	(a, c_1, c_2)	(a, c_2, c_3)	(a, c_3, c_4)
1.	(4,12,33)	(4, 33, 64)	(4, 64, 105)	(4, 105, 156)
2.	(4,33,64)	(4, 64, 105)	(4, 105, 156)	(4, 156, 217)
3.	(4,64,105)	(4, 105, 156)	(4, 156, 217)	(4, 217, 288)
4.	(4,105,156)	(4, 156, 217)	(4, 217, 288)	(4, 288, 369)

Sequence 6:

Let $a = 12, c_0 = 13k^2 - 10k + 1$

It is observed that

$$ac_0 + a + c_0 = (13k-5)^2$$

Let c_1 be any integer such that

$$(a+1)c_1 + a = \alpha^2 \tag{61}$$

$$(c_0+1)c_1 + c_0 = \beta^2 \tag{62}$$

Eliminating c_1 between (61) and (62), we have

$$(c_0+1)\alpha^2 - (a+1)\beta^2 = (a-c_0) \tag{63}$$

Introducing the linear transformations

$$\alpha = X + (a+1)T \ , \ \beta = X + (c_0+1)T \tag{64}$$

in (63) and simplifying we get

$$X^2 = (a+1)(c_0+1)T^2 - 1$$

which is satisfied by $T = 1$, $X = 13k-5$

In view of (64) and (61), it is seen that

$$c_1 = 13k^2 + 16k + 4$$

Let c_2 be any integer such that

$$(a+1)c_2 + a = \alpha^2 \tag{65}$$

$$(c_1+1)c_2 + c_1 = \beta^2 \tag{66}$$

Eliminating c_2 between (65) and (66), we have

$$(c_1+1)\alpha^2 - (a+1)\beta^2 = (a-c_1) \tag{67}$$

Introducing the linear transformations

$$\alpha = X + (a+1)T \ , \ \beta = X + (c_1+1)T \tag{68}$$

in (67) and simplifying we get

$$X^2 = (a+1)(c_1+1)T^2 - 1$$

Which is satisfied by $T=1$, $X=13k+8$

In view of (68) and (65), it is seen that

$$c_2 = 13k^2 + 42k + 33$$

Let c_3 be any integer such that

$$(a+1)c_3 + a = \alpha^2 \tag{69}$$

$$(c_2+1)c_3 + c_2 = \beta^2 \tag{70}$$

Eliminating c_3 between (69) and (70), we have

$$(c_2+1)\alpha^2 - (a+1)\beta^2 = (a-c_2) \tag{71}$$

Introducing the linear transformations

$$\alpha = X + (a+1)T \ , \ \beta = X + (c_2+1)T \tag{72}$$

In (71) and simplifying we get

$$X^2 = (a+1)(c_2+1)T^2 - 1$$

Which is satisfied by $T=1$, $X=13k+21$

In view of (72) and (69), it is seen that

$$c_3 = 13k^2 + 68k + 88$$

The repetition of the above process leads to the generation of sequence of 3-tuples whose general form is given by (a, c_{s-1}, c_s) where

$$c_{s-1} = 13k^2 + (26s-36)k + (13s^2 - 36s + 24) \ , \ s = 1,2,3,\ldots$$

A few numerical examples are presented in Table 6 below:

Table 6: Numerical Examples

k	(a, c_0, c_1)	(a, c_1, c_2)	(a, c_2, c_3)	(a, c_3, c_4)
1.	(12,33,88)	(12, 88, 169)	(12, 169, 276)	(12, 276, 409)
2.	(12,88,169)	(12, 169,276)	(12, 276, 409)	(12, 409, 568)
3.	(12,169,276)	(12, 276,409)	(12, 409, 568)	(12, 568, 753)
4.	(12,276,409)	(12,409,568)	(12, 568,753)	(12, 753, 964)

Section - III

Triples (a_1, a_2, a_3) such that $(a_i + a_j) = a\ perfect\ square\ , 1 \le i < j \le 3$

Triple: 1

Let $a = 2t_{3,2k} = 4k^2 + 2k$ and $b = 2k + 1$

$$a + b = (2k+1)^2$$

Let c be any non-zero integer such that

$$a + c = \alpha^2$$

$$b + c = \beta^2$$

Using some algebra

We have

$$c = 24k\, P_{k-1}^3 - 2k$$

Here $\left(2t_{3,2k}, 2k+1, 24k\, P_{k-1}^3 - 2k\right)$ is the required triple such that the sum of any two members is a perfect square.

Properties:

- $c - a + 2b + 2$ is a perfect square
- $c + 3a - 2b + 6$ is a perfect square
- $2a - b + c + 1 = 8k\, CP_k^3$

Triple: 2

Let $a = Ct_{10,2k} = 20k^2 + 10k + 1$ and $b = 5t_{10,2k} = 80k^2 - 30k$

$$a + b = (10k - 1)^2$$

Let c be any non-zero integer such that

$$a + c = \alpha^2$$

$$b + c = \beta^2$$

Using some algebra

We have

$$c = 100(t_{8,k})^2 - 5t_{10,2k},\ k > 1$$

Here $\left(Ct_{10,2k}, 5t_{10,2k}, 100(t_{8,k})^2 - 5t_{10,2k}\right)$ is the required triple such that the sum of any two members is a perfect square.

Properties:

- $4(a-1) - b \equiv 0 \pmod{70}$
- $3(a-1) + b \equiv 0 \pmod{140}$
- $c - 4b - 15a \equiv 0 \pmod{15}$

Triple: 3

Let $a = 8t_{3,k} = 4k^2 + 4k, k > 1$ and $b = 1$

$$a + b = (2k+1)^2$$

Let c be any non-zero integer such that

$$a + c = \alpha^2$$

$$b + c = \beta^2$$

Using some algebra

We have

$$c = 2k\,SO_k + 12CS_k^4 + 4t_{3,k-1} - 6k$$

Here $\left(8t_{3,k}, 1, 2k\ SO_k + 12CS_k^4 + 4t_{3,k-1} - 6k\right)$ is the required triple such that the sum of any two members is a perfect square.

Properties:

- $c - 2ka = 8k\,CP_k^3 - t_{25,k} - 15k$
- $2k^2a - c = 8k\,CP_k^3 - t_{10,k} + 5k$

For simplicity some more triples satisfying the required condition are given below:

Triple 4	$\left(t_{12,2n}+2t_{3,2n}+1,\, t_{8,2n}-2n,\, 6n+24nCP_{9,n}\right)$
Triple 5	$\left(t_{34,n}+t_{42,n},\, 11GNO_{n}-10,\, 4\left(t_{20,n}\right)^{2}-48nt_{20,n}-68t_{3,n-1}+142n^{2}\right)$
Triple 6	$\left(S_{n},\, 6t_{12,n}+18,\, 36n\,CP_{24,n}+78n^{2}+6n\right)$
Triple 7	$\left(4PR_{n},\, 1,\, 8nP_{n}^{5}-24CP_{n}^{3}+8n\right)$
Triple 8	$\left(7\left(6P_{n}^{3}-2P_{n}^{5}\right),\, 4t_{3,n}+4,\, 36\left(t_{3,n}\right)^{2}-28t_{3,n}\right)$

Conclusion

A few nice and beautiful patterns of 3-tuples have been illustrated in this book. The authors hope that the readers of this book would have enjoyed the formulation of the patterns of numbers apart from the patterns of numbers, namely, Arithmetic numbers, Harmonic numbers, Geometric numbers, Polygonal numbers, Pyramidal numbers, Automorphic numbers, Sphenic numbers, Fibonacci numbers and so on. In other words, it is worth mentioning here that numbers are rich in variety. Interesting readers on patterns of numbers may search for formulation of not only 3- tuples but also of higher order tuples.

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