Sequences of Gaussian Diophantine 3-Tuples

Authors

A. Vijayasankar
Assistant Professor, Department of Mathematics, National College,
Affiliated to Bharathidasan University, Trichy-620 001, Tamil Nadu, India
E-mail: avsankar70@yahoo.com

Sharadha Kumar
Research Scholar, Department of Mathematics, National College, Affiliated
to Bharathidasan University, Trichy-620 001, Tamil Nadu, India
E-mail: sharadhak12@gmail.com

M. A. Gopalan
Professor, Department of Mathematics, Shrimati Indira Gandhi College,
Affiliated to Bharathidasan University, Trichy-620 002, Tamil Nadu, India
E-mail:mayilgopalan@gmail.com

Publication Month and Year: June 2021

Pages: 32

E-BOOK ISBN: 978-81-952994-0-9

Academic Publications

C-11, 169, Sector-3, Rohini, Delhi, India

Website: www.publishbookonline.com

Email: publishbookonline@gmail.com

Phone: +91-9999744933

Sequences of Gaussian Diophantine 3-Tuples

Abstract

This paper deals with the study of constructing sequences of diophantine triples (a, b, c) through Gaussian integers such that the product of any two elements of the set added by a non-zero integer or a polynomial with integer coefficients or Gaussian integer is a perfect square.

2010 Mathematics Subject Classification: 11D99

Introduction

A theory that can be explained in a regular and systematic way is a pattern. The essence of mathematical calculations is represented by numbers and the theory of numbers can be taught as a set of patterns as it can be explained in a regular and systematic way. It is to be noted here that the number pattern is a sequence of numbers establishing a common relationship between them. Numbers exhibit fascinating, beautiful and curious varieties of patterns, namely, polygonal numbers, Fibonacci numbers, Lucas numbers, Ramanujan numbers, Kynea numbers, Jacobsthal numbers and so on. The problem of constructing the sets in distinct integers with property that the product of any two of its members is one less than a square has a very long history and such sets were studied by Diophantus. In particular, if S is a set of three non-zero distinct integers a_1, a_2, a_3 such that the product of any two members of the set with the addition of n (a non-zero integer or a polynomial with integer coefficients) is a perfect square, then S is said to be Diophantine 3-tuple with property $D(n)$. In this context one may refer [1-16].

This book concerns with the analogous problems for Gaussian integers. A Gaussian integer is a complex number of the form $a + ib, i = \sqrt{-1}$, where real and imaginary parts are non-zero integers. A set of three distinct non-zero Gaussian integers is said to be a Gaussian Diophantine 3-tuple with property D(n) if the product of any two members of set with addition of n (a non-zero integer or a polynomial with integer coefficients or Gaussian integer) is a perfect square. It seems that much work has not been done in the construction of Gaussian Diophantine 3-tuples.The construction of sequences of Gaussian Diophantine 3-tuples with suitable properties is illustrated in this book.

Method of Analysis

Sequence: 1

Let $a = \alpha + ik\beta$, $c_0 = \alpha + i(k+4)\beta$

It is observed that

$$ac_0 - 4\beta^2 = [\alpha + i(k+2)\beta]^2$$

Therefore, the pair (a, c_0) represents diophantine 2-tuple with the property $D(-4\beta^2)$.

Let c_1 be any non-zero polynomial such that

$$ac_1 - 4\beta^2 = p^2 \tag{1}$$

$$c_0 c_1 - 4\beta^2 = q^2 \tag{2}$$

Eliminating c_1 between (1) and (2), we have

$$c_0 p^2 - aq^2 = (a - c_0)4\beta^2 \tag{3}$$

Introducing the linear transformations

$$p = X + aT \ , \ q = X + c_0 T \tag{4}$$

In (3) and simplifying we get

$$X^2 = ac_0 T^2 - 4\beta^2$$

Which is satisfied by $T = 1$, $X = \alpha + i(k+2)\beta$

In view of (4) and (1), it is seen that

$$c_1 = 4\alpha + i\beta(4k+8)$$

Note that (a, c_0, c_1) represents diophantine 3-tuple with property $D(-4\beta^2)$

Taking (a, c_1) and employing the above procedure, it is seen that the triple (a, c_1, c_2) where

$$c_2 = 9\alpha + i\beta(9k+12)$$

Exhibits diophantine 3-tuple with property $D(-4\beta^2)$

Taking (a, c_2) and employing the above procedure, it is seen that the triple (a, c_2, c_3) where

$$c_3 = 16\alpha + i\beta(16k + 16)$$

Exhibits diophantine 3-tuple with property $D(-4\beta^2)$

Taking (a, c_3) and employing the above procedure, it is seen that the triple (a, c_3, c_4) where

$$c_4 = 25\alpha + i\beta(25k + 20)$$

Exhibits diophantine 3-tuple with property $D(-4\beta^2)$

The repetition of the above process leads to the generation of sequence of diophantine 3-tuples whose general form is given by $(a, c_{\partial-1}, c_{\partial})$where

$$c_{\partial-1} = \partial^2\alpha + i\beta(\partial^2 k + 4\partial),\ \partial = 1, 2, 3, ...$$

Table 1: Numerical Examples with property $D(-4\beta^2)$

k	α	β	(a,c₀,c₁)	(a,c₁,c₂)	(a,c₂,c₃)
1	3	2	(3 +i2, 3+i10,12+i24)	(3+i2, 12+i24,27+i42)	(3+i2, 27+i42,48+i64)
2	4	3	(4+i6,4+i18, 16+i48)	(4+i6,16+i48,36+i90)	(4+i6,36+i90,64+i144)
3	5	4	(5+i12, 5+i28,20+i80)	(5+i12, 20+i80,45+i156)	(5+i12,45+i156,80+i256)
4	6	5	(6+i20,6+i40,24+i120)	(6+i20,24+i120,54+i240)	(6+i20,54+i240,96+i400)
5	7	6	(7+i30, 7+i54,28+i168)	(7+i30,28+i168,63+i342)	(7+i30,63+i342,112+i576)

Sequence: 2

Let $a = \alpha + i(k+4)\beta$, $c_0 = \alpha + ik\beta$

It is observed that

$$ac_0 - 4\beta^2 = [\alpha + i(k+2)\beta]^2$$

Therefore, the pair (a, c_0) represents diophantine 2-tuple with the property $D(-4\beta^2)$.

Let c_1 be any non-zero polynomial such that

$$ac_1 - 4\beta^2 = p^2 \qquad (5)$$

$$c_0 c_1 - 4\beta^2 = q^2 \qquad (6)$$

Eliminating c_1 between (5) and (6), we have

$$c_0p^2 - aq^2 = (a - c_0)4\beta^2 \quad (7)$$

Introducing the linear transformations

$$p = X + aT \ , \ q = X + c_0T \quad (8)$$

In (7) and simplifying we get

$$X^2 = ac_0 T^2 - 4\beta^2$$

Which is satisfied by $T = 1$, $X = \alpha + i(k + 2)\beta$

In view of (8) and (5), it is seen that

$$c_1 = 4\alpha + i\beta(4k + 8)$$

Note that (a, c_0, c_1) represents diophantine 3-tuple with property $D(-4\beta^2)$

Taking (a, c_1) and employing the above procedure, it is seen that the triple (a, c_1, c_2) where

$$c_2 = 9\alpha + i\beta(9k + 24)$$

Exhibits diophantine 3-tuple with property $D(-4\beta^2)$

Taking (a, c_2) and employing the above procedure, it is seen that the triple (a, c_2, c_3) where

$$c_3 = 16\alpha + i\beta(16k + 48)$$

Exhibits diophantine 3-tuple with property $D(-4\beta^2)$

Taking (a, c_3) and employing the above procedure, it is seen that the triple (a, c_3, c_4) where

$$c_4 = 25\alpha + i\beta(25k + 80)$$

Exhibits diophantine 3-tuple with property $D(-4\beta^2)$

The repetition of the above process leads to the generation of sequence of diophantine 3-tuples whose general form is given by $(a, c_{\partial-1}, c_{\partial})$where

$$c_{\partial-1} = \partial^2\alpha + i\beta(\partial^2k + 4\partial(\partial - 1)), \ \partial = 1, 2, 3, ...$$

Table 2: Numerical Examples with property $D(-4\beta^2)$

k	α	β	**(a,c₀, c₁)**	**(a, c₁,c₂)**	**(a,c₂,c₃)**
1	3	2	(3+i10,3+i2,12+i24)	(3+i10,12+i24,27+i66)	(3+i10,27+i66, 48+i128)
2	4	3	(4+i18, 4+i6,16+i48)	(4+i18,16+i48,36+i126)	(4+i18, 36+i126,64+i240)
3	5	4	(5+i28,5+i12,20+i80)	(5+i28,20+i80,45+i204)	(5+i28,45+i204,80+i384)
4	6	5	(6+i40,6+i20,24+i120)	(6+i40, 24+i120,54+i300)	(6+i40,54+i300,96+i560)
5	7	6	(7+i54,7+i30,28+i168)	(7+i54, 28+i168,63+i414)	(7+i54,63+i414,112+i768)

Sequence: 3

Let $a=\alpha+ik\beta$, $c_0=\alpha+i\beta(k+3)$

It is observed that

$$ac_0+i\alpha\beta-(k+4)\beta^2=[\alpha+i(k+2)\beta]^2$$

Therefore, the pair (a,c_0) represents diophantine 2-tuple with the property $D(i\alpha\beta-(k+4)\beta^2)$.

Let c_1 be any non-zero polynomial such that

$$ac_1+i\alpha\beta-(k+4)\beta^2=p^2 \qquad (9)$$

$$c_0c_1+\alpha\beta-(k+4)\beta^2=q^2 \qquad (10)$$

Eliminating c_1 between (9) and (10), we have

$$c_0p^2-aq^2=(c_0-a)(i\alpha\beta-(k+4)\beta^2) \qquad (11)$$

Introducing the linear transformations

$$p=X+aT \; , \; q=X+c_0T \qquad (12)$$

In (11) and simplifying we get

$$X^2=ac_0T^2+i\alpha\beta-(k+4)\beta^2$$

Which is satisfied by $T=1$, $X=\alpha+i(k+2)\beta$

In view of (12) and (9), it is seen that

$$c_1=4\alpha+i\beta(4k+7)$$

Note that (a,c_0,c_1) represents diophantine 3-tuple with property $D(i\alpha\beta-(k+4)\beta^2)$

Taking (a, c_1) and employing the above procedure, it is seen that the triple (a, c_1, c_2) where

$$c_2 = 9\alpha + i\beta(9k + 11)$$

Exhibits diophantine 3-tuple with property $D(i\alpha\beta - (k+4)\beta^2)$

Taking (a, c_2) and employing the above procedure, it is seen that the triple (a, c_2, c_3) where

$$c_3 = 16\alpha + i\beta(16k + 15)$$

Exhibits diophantine 3-tuple with property $D(i\alpha\beta - (k+4)\beta^2)$

Taking (a, c_3) and employing the above procedure, it is seen that the triple (a, c_3, c_4) where

$$c_4 = 25\alpha + i\beta(25k + 19)$$

Exhibits diophantine 3-tuple with property $D(i\alpha\beta - (k+4)\beta^2)$

The repetition of the above process leads to the generation of sequence of diophantine 3-tuples whose general form is given by $(a, c_{\partial-1}, c_{\partial})$where

$$c_{\partial-1} = \partial^2\alpha + i\beta(\partial^2 k + 4\partial - 1), \ \partial = 1, 2, 3, \ldots$$

Table 3: Numerical Examples with property $D(i\alpha\beta - (k+4)\beta^2)$

k	α	β	**(a,c_0,c_1)**	**(a,c_1,c_2)**	**(a,c_2,c_3)**
1	3	2	(3 +i2,3 +i8,12+i22)	(3+i2,12+i22,27+i40)	(3+i2,27+i40,48+i62)
2	4	3	(4+i6,4+i15,16 +i45)	(4 +i6, 16+i45, 36+i87)	(4+i6,36+i87,64+i141)
3	5	4	(5+il2,5+i24,20+i76)	(5+i12,20+i76,45+i152)	(5+i12,45+i152,80+i252)
4	6	5	(6+i20,6+i35,24+ill5)	(6+i20,24+i115,54+i235)	(6+i20,54+i235,96+i395)
5	7	6	(7+i30,7+i48,28+i162)	(7+i30,28+i162,63+i336)	(7+i30,63+i336,112+i570)

Sequence: 4

Let $a = \alpha + i\beta(k+3), \ c_0 = \alpha + ik\beta$

It is observed that

$$ac_0 + i\alpha\beta - (k+4)\beta^2 = [\alpha + i(k+2)\beta]^2$$

Therefore, the pair (a, c_0) represents diophantine 2-tuple with the property $D(i\alpha\beta - (k+4)\beta^2)$.

Let c_1 be any non-zero polynomial such that

$$ac_1 + i\alpha\beta - (k+4)\beta^2 = p^2 \tag{13}$$

$$c_0c_1 + \alpha\beta - (k+4)\beta^2 = q^2 \tag{14}$$

Eliminating c_1 between (13) and (14), we have

$$c_0p^2 - aq^2 = (c_0 - a)\left(i\alpha\beta - (k+4)\beta^2\right) \tag{15}$$

Introducing the linear transformations

$$p = X + aT \ , \ q = X + c_0T \tag{16}$$

In (15) and simplifying we get

$$X^2 = ac_0 T^2 + i\alpha\beta - (k+4)\beta^2$$

Which is satisfied by $T = 1$, $X = \alpha + i(k+2)\beta$

In view of (16) and (13), it is seen that

$$c_1 = 4\alpha + i\beta(4k+7)$$

Note that (a, c_0, c_1) represents diophantine 3-tuple with property $D(i\alpha\beta - (k+4)\beta^2)$

Taking (a, c_1) and employing the above procedure, it is seen that the triple (a, c_1, c_2) where

$$c_2 = 9\alpha + i\beta(9k+20)$$

Exhibits diophantine 3-tuple with property $D(i\alpha\beta - (k+4)\beta^2)$

Taking (a, c_2) and employing the above procedure, it is seen that the triple (a, c_2, c_3) where

$$c_3 = 16\alpha + i\beta(16k+39)$$

Exhibits diophantine 3-tuple with property $D(i\alpha\beta - (k+4)\beta^2)$

Taking (a, c_3) and employing the above procedure, it is seen that the triple (a, c_3, c_4) where

$$c_4 = 25\alpha + i\beta(25k+64)$$

Exhibits diophantine 3-tuple with property $D(i\alpha\beta - (k+4)\beta^2)$

The repetition of the above process leads to the generation of sequence of diophantine 3-tuples whose general form is given by $(a, c_{\partial-1}, c_{\partial})$ where

$$c_{\partial-1} = \partial^2\alpha + i\beta((k+3)\partial^2 - 2\partial - 1), \quad \partial = 1, 2, 3, ...$$

Table 4: Numerical Examples with property $D(i\alpha\beta - (k+4)\beta^2)$

k	α	β	(a,c₀,c₁)	(a,c₁,c₂)	(a,c₂,c₃)
1	3	2	(3+i8, 3+i2,12+i22)	(3+i8,12+i22,27+i58)	(3+i8,27+i58,48+i110)
2	4	3	(4+i15,4+i6,16+i45)	(4+i15,16+i45,36+i114)	(4+i15,36+i114,64+i213)
3	5	4	(5+i24,5+i12,20+i76)	(5+i24,20+i76,45+i188)	(5+i24,45+i188,80+i348)
4	6	5	(6+i35,6+i20,24+i115)	(6+i35,24+i115,54+i280)	(6+i35,54+i280,96+i515)
5	7	6	(7+i48,7+i30,28+i162)	(7+i48,28+i162,63+i390)	(7+i48,63+i390,112+i714)

Sequence: 5

Let $a = \alpha + i(k+1)\beta, \quad c_0 = \alpha - i(k-1)\beta$

It is observed that

$$ac_0 - k^2\beta^2 = (\alpha + i\beta)^2$$

Therefore, the pair (a, c_0) represents diophantine 2-tuple with the property $D(-k^2\beta^2)$.

Let c_1 be any non-zero polynomial such that

$$ac_1 - k^2\beta^2 = p^2 \tag{17}$$

$$c_0c_1 - k^2\beta^2 = q^2 \tag{18}$$

Eliminating c_1 between (17) and (18), we have

$$c_0p^2 - aq^2 = (a - c_0)(-k^2\beta^2) \tag{19}$$

Introducing the linear transformations

$$p = X + aT \ , \ q = X + c_0T \tag{20}$$

In (19) and simplifying we get

$$X^2 = ac_0T^2 - k^2\beta^2$$

Which is satisfied by $T = 1$, $X = \alpha + i\beta$

In view of (20) and (17), it is seen that

$$c_1 = 4\alpha + i4\beta$$

Note that (a, c_0, c_1) represents diophantine 3-tuple with property $D(-k^2\beta^2)$

Taking (a, c_1) and employing the above procedure, it is seen that the triple (a, c_1, c_2) where

$$c_2 = 9\alpha + i\beta(3k + 9)$$

Exhibits diophantine 3-tuple with property $D(-k^2\beta^2)$

Taking (a, c_2) and employing the above procedure, it is seen that the triple (a, c_2, c_3) where

$$c_3 = 16\alpha + i\beta(8k + 16)$$

Exhibits diophantine 3-tuple with property $D(-k^2\beta^2)$

Taking (a, c_3) and employing the above procedure, it is seen that the triple (a, c_3, c_4) where

$$c_4 = 25\alpha + i\beta(15k + 25)$$

Exhibits diophantine 3-tuple with property $D(-k^2\beta^2)$

The repetition of the above process leads to the generation of sequence of diophantine 3-tuples whose general form is given by $(a, c_{\partial-1}, c_\partial)$where

$$c_{\partial-1} = \partial^2\alpha + i\beta((k+1)\partial^2 - 2k\partial),\ \partial = 1, 2, 3, ...$$

Table 5: Numerical Examples with property $D(-k^2\beta^2)$

k	α	β	**(a,c₀,c₁)**	**(a,c₁,c₂)**	**(a,c₂,c3)**
2	3	2	(3+i6, 3-i2, 12+i8)	(3+i6, 12+i8, 27+i30)	(3+i6, 27+i30,48+i64)
2	4	3	(4+i9, 4-i3, 16+i12)	(4+i9, 16+i12, 36+i45)	(4+i9, 36+i45, 64+i96)
3	5	4	(5+i6,5-i8,20+il6)	(5+i16, 20+i16, 45+i72)	(5+i16, 45+i72,80+i160)
4	6	5	(6+i25, 6-i15, 24+i20)	(6+i25, 24+i20, 54+i105)	(6+i25, 54+i105, 96+i240)
5	7	6	(7+i36,7-i24,28+i24)	(7+i36, 28+i24,63+i144)	(7+i36, 63+i144, 112+i336)

Sequence: 6

Let $a=\alpha-i(k-1)\beta$, $c_0=\alpha+i(k+1)\beta$

It is observed that

$$ac_0-k^2\beta^2=(\alpha+i\beta)^2$$

Therefore, the pair (a,c_0) represents diophantine 2-tuple with the property $D(-k^2\beta^2)$.

Let c_1 be any non-zero polynomial such that

$$ac_1-k^2\beta^2=p^2 \tag{21}$$

$$c_0c_1-k^2\beta^2=q^2 \tag{22}$$

Eliminating c_1 between (21) and (22), we have

$$c_0p^2-aq^2=(a-c_0)(-k^2\beta^2) \tag{23}$$

Introducing the linear transformations

$$p=X+aT\ ,\ q=X+c_0T \tag{24}$$

In (23) and simplifying we get

$$X^2=ac_0T^2-k^2\beta^2$$

Which is satisfied by $T=1$, $X=\alpha+i\beta$

In view of (24) and (21), it is seen that

$$c_1=4\alpha+i4\beta$$

Note that (a,c_0,c_1) represents diophantine 3-tuple with property $D(-k^2\beta^2)$

Taking (a,c_1) and employing the above procedure, it is seen that the triple (a,c_1,c_2) where

$$c_2=9\alpha+i\beta(9-3k)$$

Exhibits diophantine 3-tuple with property $D(-k^2\beta^2)$

Taking (a,c_2) and employing the above procedure, it is seen that the triple (a,c_2,c_3) where

$$c_3 = 16\alpha + i\beta(16 - 8k)$$

Exhibits diophantine 3-tuple with property $D(-k^2\beta^2)$

Taking (a, c_3) and employing the above procedure, it is seen that the triple (a, c_3, c_4) where

$$c_4 = 25\alpha + i\beta(25 - 15k)$$

Exhibits diophantine 3-tuple with property $D(-k^2\beta^2)$

The repetition of the above process leads to the generation of sequence of diophantine 3-tuples whose general form is given by $(a, c_{\partial-1}, c_\partial)$where

$$c_{\partial-1} = \partial^2\alpha + i\beta((1-k)\partial^2 + 2k\partial),\ \partial = 1, 2, 3, \ldots$$

Table 6: Numerical Examples with property $D(-k^2\beta^2)$

k	α	β	(a,c₀,c₁)	(a,c₁,c₂)	(a,c₂,c₃)
5	3	2	(3-i8, 3+i12,12+i8)	(3-i8,12+i8,27-i12)	(3-i8,27-i12,48-i48)
7	4	3	(4-i18, 4+i24,16+i12)	(4-i18,16+i12,36-i36)	(4-i18,36-i36,64-i120)
20	5	4	(5-i76,5+i84,20+i16)	(5-i76,20+i16,45-i204)	(5-i76,45-i204,80-i576)
4	6	5	(6-i15,6+i25,24+i20)	(6-i15,24+i20,54-i15)	(6-i15,54-i15,96-i80)
5	7	6	(7-i24,7+i36,28+i24)	(7-i24,28+i24,63-i36)	(7-i24, 63-i36,112-i144)

Sequence: 7

Let $a = \alpha + i\beta,\ \ c_0 = \alpha + 2s + i\beta$

It is observed that

$$ac_0 + s^2 = (\alpha + s + i\beta)^2$$

Therefore, the pair (a, c_0) represents diophantine 2-tuple with the property $D(s^2)$.

Let c_1 be any non-zero polynomial such that

$$ac_1 + s^2 = p^2 \tag{25}$$

$$c_0c_1 + s^2 = q^2 \tag{26}$$

Eliminating c_1 between (25) and (26), we have

$$c_0p^2 - aq^2 = (c_0 - a)(s^2) \tag{27}$$

Introducing the linear transformations

$$p = X + aT \ , \ q = X + c_0 T \tag{28}$$

In (27) and simplifying we get

$$X^2 = ac_0 T^2 + s^2$$

Which is satisfied by $T = 1 \ , \ X = \alpha + s + i\beta$

In view of (28) and (25), it is seen that

$$c_1 = 4(\alpha + i\beta) + 4s$$

Note that (a, c_0, c_1) represents diophantine 3-tuple with property $D(s^2)$

Taking (a, c_1) and employing the above procedure, it is seen that the triple (a, c_1, c_2) where

$$c_2 = 9(\alpha + i\beta) + 6s$$

Exhibits diophantine 3-tuple with property $D(s^2)$

Taking (a, c_2) and employing the above procedure, it is seen that the triple (a, c_2, c_3) where

$$c_3 = 16(\alpha + i\beta) + 8s$$

Exhibits diophantine 3-tuple with property $D(s^2)$

Taking (a, c_3) and employing the above procedure, it is seen that the triple (a, c_3, c_4) where

$$c_4 = 25(\alpha + i\beta) + 10s$$

Exhibits diophantine 3-tuple with property $D(s^2)$

The repetition of the above process leads to the generation of sequence of diophantine 3-tuples whose general form is given by $(a, c_{\partial-1}, c_{\partial})$where

$$c_{\partial-1} = \partial^2(\alpha + i\beta) + 2\partial s, \ \partial = 1, 2, 3, ...$$

Table 7: Numerical Examples with property $D(s^2)$

k	α	β	s	(a,c₀,c₁)	(a,c₁,c₂)	(a,c₂,c₃)
1	3	2	4	(3 +i2,11+i2,28+i8)	(3+i2,28+i8,51+i18)	(3+i2, 51+i18,80+i32)
2	4	3	5	(4+i3,14+i3,36+i12)	(4+i3,36+i12,66+i27)	(4+i3,66+i27,104+i48)
3	5	4	6	(5+i4,17+i4,44+i16)	(5+i4,44+i16,81+i36)	(5+i4,81+i36,128+i64)
4	6	5	7	(6+i5,20+i5,52+i20)	(6+i5,52+i20,96+i45)	(6+i5,96+i45,152+i80)
5	7	6	8	(7+i6,23+i6,60+i24)	(7+i6,60+i24,111+i54)	(7+i6,111+i54,176+i96)

Sequence: 8

Let $a = \alpha + i\beta + 2s,\ \ c_0 = \alpha + i\beta$

It is observed that

$$ac_0 + s^2 = (\alpha + s + i\beta)^2$$

Therefore, the pair (a, c_0) represents diophantine 2-tuple with the property $D(s^2)$.

Let c_1 be any non-zero polynomial such that

$$ac_1 + s^2 = p^2 \tag{29}$$

$$c_0c_1 + s^2 = q^2 \tag{30}$$

Eliminating c_1 between (29) and (30), we have

$$c_0p^2 - aq^2 = (c_0 - a)(s^2) \tag{31}$$

Introducing the linear transformations

$$p = X + aT\ ,\ q = X + c_0T \tag{32}$$

In (31) and simplifying we get

$$X^2 = ac_0T^2 + s^2$$

Which is satisfied by $T = 1\ ,\ X = \alpha + s + i\beta$

In view of (32) and (29), it is seen that

$$c_1 = 4(\alpha + i\beta) + 4s$$

Note that (a, c_0, c_1) represents diophantine 3-tuple with property $D(s^2)$

Taking (a, c_1) and employing the above procedure, it is seen that the triple (a, c_1, c_2) where

$$c_2 = 9(\alpha + i\beta) + 12s$$

Exhibits diophantine 3-tuple with property $D(s^2)$

Taking (a, c_2) and employing the above procedure, it is seen that the triple (a, c_2, c_3) where

$$c_3 = 16(\alpha + i\beta) + 24s$$

Exhibits diophantine 3-tuple with property $D(s^2)$

Taking (a, c_3) and employing the above procedure, it is seen that the triple (a, c_3, c_4) where

$$c_4 = 25(\alpha + i\beta) + 40s$$

Exhibits diophantine 3-tuple with property $D(s^2)$

The repetition of the above process leads to the generation of sequence of diophantine 3-tuples whose general form is given by $(a, c_{\partial-1}, c_{\partial})$where

$$c_{\partial-1} = \partial^2(\alpha + i\beta) + (2\partial^2 - 2\partial)s, \ \partial = 1, 2, 3, ...$$

Table 8: Numerical Examples with property $D(s^2)$

k	α	β	s	(a,c₀,c₁)	(a,c₁,c₂)	(a,c₂,c₃)
1	3	2	4	(11+i2, 3+i2, 28+i8)	(11+i2, 28+i8,75+i18)	(11+i2,75+i18,144+i32)
2	4	3	5	(14+i3, 4+i3,36+i12)	(14+i3, 36+i12, 96+i27)	(14+i3, 96+i27,184+i48)
3	5	4	6	(1+i4,5+i4,44+i16)	(17+i4,44+i16, 117+i36)	(17+i4,117+i36,224+i64)
4	6	5	7	(20+i5, 6+i5, 52+i20)	(20+i5, 52+i20, 138+i45)	(20+i5,138+i45,264+i80)
5	7	6	8	(23+i6,7+i6,60+i24)	(23+i6,60+i24, 159+i54)	(23+i6,159+i54,304+i96)

Sequence: 9

Let $a = \alpha + i\beta, \ c_0 = \alpha + i(k+1)\beta$

It is observed that

$$ac_0 + (k^2 + k - 1)\beta^2 - i(2k-1)\beta\alpha = (\alpha + i\beta)^2$$

Therefore, the pair (a, c_0) represents diophantine 2-tuple with the property $D\left[\left(k^2+k-1\right)\beta^2-i\left(2k-1\right)\beta\alpha\right]$.

Let c_1 be any non-zero polynomial such that

$$ac_1+\left(k^2+k-1\right)\beta^2-i\left(2k-1\right)\beta\alpha=p^2 \quad (33)$$

$$c_0c_1+\left(k^2+k-1\right)\beta^2-i\left(2k-1\right)\beta\alpha=q^2 \quad (34)$$

Eliminating c_1 between (33) and (34), we have

$$c_0p^2-aq^2=\left(c_0-a\right)\left(\left(k^2+k-1\right)\beta^2-i\left(2k-1\right)\beta\alpha\right) \quad (35)$$

Introducing the linear transformations

$$p=X+aT\ ,\ q=X+c_0T \quad (36)$$

In (35) and simplifying we get

$$X^2=ac_0\,T^2+\left(k^2+k-1\right)\beta^2-i\left(2k-1\right)\beta\alpha$$

Which is satisfied by $T=1\ ,\ X=\alpha+i\beta$

In view of (36) and (33), it is seen that

$$c_1=4\alpha+i\beta(2k+3)$$

Note that $\left(a,c_0,c_1\right)$ represents diophantine 3-tuple with property $D\left[\left(k^2+k-1\right)\beta^2-i\left(2k-1\right)\beta\alpha\right]$

Taking $\left(a,c_1\right)$ and employing the above procedure, it is seen that the triple $\left(a,c_1,c_2\right)$ where

$$c_2=9\alpha+i\beta(5k+5)$$

Exhibits diophantine 3-tuple with property $D\left[\left(k^2+k-1\right)\beta^2-i\left(2k-1\right)\beta\alpha\right]$

Taking $\left(a,c_2\right)$ and employing the above procedure, it is seen that the triple $\left(a,c_2,c_3\right)$ where

$$c_3=16\alpha+i\beta(10k+7)$$

Exhibits diophantine 3-tuple with property $D\left[\left(k^2+k-1\right)\beta^2-i\left(2k-1\right)\beta\alpha\right]$

Taking $\left(a,c_3\right)$ and employing the above procedure, it is seen that the triple (a, c_3, c_4) where

$$c_4 = 25\alpha + i\beta(17k+9)$$

Exhibits diophantine 3-tuple with property $D[(k^2+k-1)\beta^2 - i(2k-1)\beta\alpha]$

The repetition of the above process leads to the generation of sequence of diophantine 3-tuples whose general form is given by $(a, c_{\partial-1}, c_\partial)$ where

$$c_{\partial-1} = \partial^2\alpha + i\beta\left(k\partial^2 - (2k-2)\partial + 2k-1\right),\ \partial = 1,2,3,...$$

Table 9: Numerical Examples with property $D[(k^2+k-1)\beta^2 - i(2k-1)\beta\alpha]$

k	α	β	(a,c₀,c₁)	(a, c₁, c₂)	(a,c₂,c₃)
1	3	2	(3+i2,3+i4,12+i10)	(3+i2,12+i10,27+i20)	(3+i2,27+i20,48+i34)
2	4	3	(4+i3,4+i9,16+i21)	(4+i3,16+i21,36+i45)	(4+i3,36+i45,45+i80)
3	5	4	(5+i4,5+i16,20+i36)	(5+i4,20+i36,45+i80)	(5+i4,45+i80,80+i148)
4	6	5	(6+i5,6+i25,24+i55)	(6+i5,24+i55,54+i125)	(6+i5,54+i125,96+i235)
5	7	6	(7+i6,7+i36,28+i78)	(7+i6,28+i78,63+i180)	(7+i6,63+i180,112+i342)

Sequence: 10

Let $a = \alpha + i(k+1)\beta,\ \ c_0 = \alpha + i\beta$

It is observed that

$$ac_0 + (k^2+k-1)\beta^2 - i(2k-1)\beta\alpha = (\alpha + i\beta)^2$$

Therefore, the pair (a, c_0) represents diophantine 2-tuple with the property $D[(k^2+k-1)\beta^2 - i(2k-1)\beta\alpha]$.

Let c_1 be any non-zero polynomial such that

$$ac_1 + (k^2+k-1)\beta^2 - i(2k-1)\beta\alpha = p^2 \quad (37)$$

$$c_0c_1 + (k^2+k-1)\beta^2 - i(2k-1)\beta\alpha = q^2 \quad (38)$$

Eliminating c_1 between (37) and (38), we have

$$c_0p^2 - aq^2 = (c_0 - a)\left((k^2+k-1)\beta^2 - i(2k-1)\beta\alpha\right) \quad (39)$$

Introducing the linear transformations

$$p = X + aT\ ,\ \ q = X + c_0T \quad (40)$$

In (39) and simplifying we get

$$X^2 = ac_0 T^2 + (k^2 + k - 1)\beta^2 - i(2k-1)\beta\alpha$$

Which is satisfied by $T = 1$, $X = \alpha + i\beta$

In view of (40) and (37), it is seen that

$$c_1 = 4\alpha + i\beta(2k+3)$$

Note that (a, c_0, c_1) represents diophantine 3-tuple with property $D[(k^2 + k - 1)\beta^2 - i(2k-1)\beta\alpha]$

Taking (a, c_1) and employing the above procedure, it is seen that the triple (a, c_1, c_2) where

$$c_2 = 9\alpha + i\beta(5k+8)$$

Exhibits diophantine 3-tuple with property $D[(k^2 + k - 1)\beta^2 - i(2k-1)\beta\alpha]$

Taking (a, c_2) and employing the above procedure, it is seen that the triple (a, c_2, c_3) where

$$c_3 = 16\alpha + i\beta(10k+15)$$

Exhibits diophantine 3-tuple with property $D[(k^2 + k - 1)\beta^2 - i(2k-1)\beta\alpha]$

Taking (a, c_3) and employing the above procedure, it is seen that the triple (a, c_3, c_4) where

$$c_4 = 25\alpha + i\beta(17k+24)$$

Exhibits diophantine 3-tuple with property $D[(k^2 + k - 1)\beta^2 - i(2k-1)\beta\alpha]$

The repetition of the above process leads to the generation of sequence of diophantine 3-tuples whose general form is given by $(a, c_{\partial-1}, c_{\partial})$where

$$c_{\partial-1} = \partial^2\alpha + i\beta\ ((k+1)\partial^2 - 2k\partial + 2k - 1),\ \partial = 1, 2, 3, ...$$

Table 10: Numerical Examples with property $D\left[\left(k^2+k-1\right)\beta^2 - i\left(2k-1\right)\beta\alpha\right]$

k	α	β	(a,c₀,c₁)	(a, c₁,c₂)	(a, c₂,c₃)
1	3	2	(3+i4,3+i2, 12+i10)	(3+i4,12+i10,27+i26)	(3+i4,27+i26,48+i50)
2	4	3	(4+i9,4+i3,16+i21)	(4+i9,16+i21,36+i54)	(4+i9,36+i54,64+i105)
3	5	4	(5+i16,5+i4,20+i36)	(5+i16,20+i36,45+i92)	(5+i16,45+i92,80+i180)
4	6	5	(6+i25,6+i5,24+i55)	(6+i25,24+i55,54+i140)	(6+i25,54+i140,96+i275)
5	7	6	(7+i36,7+i6,28+i78)	(7+i36,28+i78,63+i198)	(7+i36,63+i198,112+i390)

Sequence: 11

Let $a = \alpha + ik^2\beta, \quad c_0 = \alpha + i(k+1)^2\beta$

It is observed that

$$ac_0 - i\alpha\beta = \left(\alpha + ik(k+1)\beta\right)^2$$

Therefore, the pair (a, c_0) represents diophantine 2-tuple with the property $D(-i\alpha\beta)$.

Let c_1 be any non-zero polynomial such that

$$ac_1 - i\alpha\beta = p^2 \quad (41)$$

$$c_0c_1 - i\alpha\beta = q^2 \quad (42)$$

Eliminating c_1 between (41) and (42), we have

$$c_0p^2 - aq^2 = (a - c_0)(i\alpha\beta) \quad (43)$$

Introducing the linear transformations

$$p = X + aT \ , \ q = X + c_0T \quad (44)$$

In (43) and simplifying we get

$$X^2 = ac_0\,T^2 - i\alpha\beta$$

Which is satisfied by $T = 1$, $X = \alpha + ik(k+1)\beta$

In view of (44) and (41), it is seen that

$$c_1 = 4\alpha + i\beta(2k+1)^2$$

Note that (a, c_0, c_1) represents diophantine 3-tuple with property $D(-i\alpha\beta)$

Taking (a, c_1) and employing the above procedure, it is seen that the triple (a, c_1, c_2) where

$$c_2 = 9\alpha + i\beta(3k+1)^2$$

Exhibits diophantine 3-tuple with property $D(-i\alpha\beta)$

Taking (a, c_2) and employing the above procedure, it is seen that the triple (a, c_2, c_3) where

$$c_3 = 16\alpha + i\beta(4k+1)^2$$

Exhibits diophantine 3-tuple with property $D(-i\alpha\beta)$

Taking (a, c_3) and employing the above procedure, it is seen that the triple (a, c_3, c_4) where

$$c_4 = 25\alpha + i\beta(5k+1)^2$$

Exhibits diophantine 3-tuple with property $D(-i\alpha\beta)$

The repetition of the above process leads to the generation of sequence of diophantine 3-tuples whose general form is given by $(a, c_{\partial-1}, c_{\partial})$where

$$c_{\partial-1} = \partial^2\alpha + i\beta\ (\partial k+1)^2,\ \ \partial = 1, 2, 3, ...$$

Table 11: Numerical Examples with property $D(-i\alpha\beta)$

k	α	β	(a,c_0,c_1)	(a, c_1,c_2)	(a,c_2,c_3)
1	3	2	(3+i2, 3+i8,12+i18)	(3+i2,12+i18,27+i32)	(3+i2,27+i32,48+i50)
2	4	3	(4+i12,4 +i27,16 +i75)	(4+i12,16+i75,36+i147)	(4+i12, 36+i147,64+i243)
3	5	4	(5+i36,5+i64,20+i196)	(5+i36,20+i196,45+i400)	(5+i36,45+i400,80+i676)
4	6	5	(6+i80,6+i125,24+i405)	(6+i80,24+i405,54+i845)	(6+i80,54+i845,96+i1445)
5	7	6	(7+i150,7+i216,28+i726)	(7+i150,28+i726,63+i1536)	(7+i150,63+i1536,112+i2646)

Sequence: 12

Let $a = \alpha + i(k+1)^2\beta,\ \ c_0 = \alpha + ik^2\beta$

It is observed that

$$ac_0 - i\alpha\beta = (\alpha + ik(k+1)\beta)^2$$

Therefore, the pair (a, c_0) represents diophantine 2-tuple with the property $D(-i\alpha\beta)$.

Let c_1 be any non-zero polynomial such that

$$ac_1 - i\alpha\beta = p^2 \qquad (45)$$

$$c_0c_1 - i\alpha\beta = q^2 \qquad (46)$$

Eliminating c_1 between (45) and (46), we have

$$c_0p^2 - aq^2 = (a - c_0)(i\alpha\beta) \qquad (47)$$

Introducing the linear transformations

$$p = X + aT \ , \ q = X + c_0T \qquad (48)$$

In (47) and simplifying we get

$$X^2 = ac_0 T^2 - i\alpha\beta$$

Which is satisfied by $T = 1$, $X = \alpha + ik(k+1)\beta$

In view of (48) and (45), it is seen that

$$c_1 = 4\alpha + i\beta(2k+1)^2$$

Note that (a, c_0, c_1) represents diophantine 3-tuple with property $D(-i\alpha\beta)$

Taking (a, c_1) and employing the above procedure, it is seen that the triple (a, c_1, c_2) where

$$c_2 = 9\alpha + i\beta(3k+2)^2$$

Exhibits diophantine 3-tuple with property $D(-i\alpha\beta)$

Taking (a, c_2) and employing the above procedure, it is seen that the triple (a, c_2, c_3) where

$$c_3 = 16\alpha + i\beta(4k+3)^2$$

Exhibits diophantine 3-tuple with property $D(-i\alpha\beta)$

Taking (a, c_3) and employing the above procedure, it is seen that the triple (a, c_3, c_4) where

$$c_4 = 25\alpha + i\beta(5k+4)^2$$

Exhibits diophantine 3-tuple with property $D(-i\alpha\beta)$

The repetition of the above process leads to the generation of sequence of diophantine 3-tuples whose general form is given by $(a, c_{\partial-1}, c_{\partial})$ where

$$c_{\partial-1} = \partial^2\alpha + i\beta\,(\partial(k+1)-1)^2,\ \partial = 1, 2, 3, ...$$

Table 12: Numerical Examples with property $D(-i\alpha\beta)$

k	α	β	**(a,c₀,c₁)**	**(a,c₁,c₂)**	**(a,c₂,c₃)**
1	3	2	(3+i8,3+i2,12+i18)	(3+i8,12+i18,27+i50)	(3+i8,27+i50,48+i98)
2	4	3	(4+i27,4+i12,16+i75)	(4+i27,16+i75,36+i192)	(4+i27,36+i192,64+i363)
3	5	4	(5+i64,5+i36,20+i196)	(5+i64,20+i196,45+i484)	(5+i64,45+i484,80+i900)
4	6	5	(6+i125,6+i80,24+i405)	(6+i125,24+i405,54+i980)	(6+i125,54+i980,96+i1805)
5	7	6	(7+i216,7+i150,28+i726)	(7+i216,28+i726,63+i1734)	(7+i216,63+i1734,112+i3174)

Sequence: 13

Let $a = \alpha + i2k\beta,\ \ c_0 = \alpha + i(2k+2)\beta$

It is observed that

$$ac_0 - \beta^2 = (\alpha + i(2k+1)\beta)^2$$

Therefore, the pair (a, c_0) represents diophantine 2-tuple with the property $D(-\beta^2)$.

Let c_1 be any non-zero polynomial such that

$$ac_1 - \beta^2 = p^2 \quad (49)$$

$$c_0c_1 - \beta^2 = q^2 \quad (50)$$

Eliminating c_1 between (49) and (50), we have

$$c_0p^2 - aq^2 = (a - c_0)\beta^2 \quad (51)$$

Introducing the linear transformations

$$p = X + aT\ ,\ q = X + c_0T \quad (52)$$

In (51) and simplifying we get

$$X^2 = ac_0\,T^2 - \beta^2$$

Which is satisfied by $T=1$, $X=\alpha+i(2k+1)\beta$

In view of (52) and (49), it is seen that

$$c_1=4\alpha+i\beta(8k+4)$$

Note that (a,c_0,c_1) represents diophantine 3-tuple with property $D(-\beta^2)$

Taking (a,c_1) and employing the above procedure, it is seen that the triple (a,c_1,c_2) where

$$c_2=9\alpha+i\beta(18k+6)$$

Exhibits diophantine 3-tuple with property $D(-\beta^2)$

Taking (a,c_2) and employing the above procedure, it is seen that the triple (a,c_2,c_3) where

$$c_3=16\alpha+i\beta(32k+8)$$

Exhibits diophantine 3-tuple with property $D(-\beta^2)$

Taking (a,c_3) and employing the above procedure, it is seen that the triple (a,c_3,c_4) where

$$c_4=25\alpha+i\beta(50k+10)$$

Exhibits diophantine 3-tuple with property $D(-\beta^2)$

The repetition of the above process leads to the generation of sequence of diophantine 3-tuples whose general form is given by $(a,c_{\partial-1},c_{\partial})$where

$$c_{\partial-1}=\partial^2\alpha+i\beta\left(2k\partial^2+2\partial\right),\ \partial=1,2,3,...$$

Table 13: Numerical Examples with property $D(-\beta^2)$

k	α	β	**(a, c₀, c₁)**	**(a, c₁, c₂,)**	**(a,c₂,c₃)**
1	3	2	(3+i4, 3+i8, 12+i24)	(3+i4, 12+i24, 27+i48)	(3+i4, 27+i48, 48+i80)
2	4	3	(4+i12, 4+i18, 16+i60)	(4+i12, 16+i60, 36+i126)	(4+i12, 36+i126, 64+i216)
3	5	4	(5+i24, 5+i32, 20+i112)	(5+i24, 20+i112, 45+i240)	(5+i24, 45+i240, 80+i416)
4	6	5	(6+i40, 6+i50, 24+i180)	(6+i40, 24+i180, 54+i390)	(6+i40, 54+i390, 96+i680)
5	7	6	(7+i60, 7+i72, 28+i264)	(7+i60, 28+i264, 63+i576)	(7+i60, 63+i576, 112+i1008)

Sequence 14:

Let $a=\alpha+i(2k+2)\beta,\ \ c_0=\alpha+i2k\beta$

It is observed that

$$ac_0-\beta^2=(\alpha+i(2k+1)\beta)^2$$

Therefore, the pair (a,c_0) represents diophantine 2-tuple with the property $D(-\beta^2)$.

Let c_1 be any non-zero polynomial such that

$$ac_1-\beta^2=p^2 \quad (53)$$

$$c_0c_1-\beta^2=q^2 \quad (54)$$

Eliminating c_1 between (53) and (54), we have

$$c_0p^2-aq^2=(a-c_0)\beta^2 \quad (55)$$

Introducing the linear transformations

$$p=X+aT\ ,\ q=X+c_0T \quad (56)$$

In (55) and simplifying we get

$$X^2=ac_0\,T^2-\beta^2$$

Which is satisfied by $T=1\ ,\ X=\alpha+i(2k+1)\beta$

In view of (56) and (53), it is seen that

$$c_1=4\alpha+i\beta(8k+4)$$

Note that (a,c_0,c_1) represents diophantine 3-tuple with property $D(-\beta^2)$

Taking (a,c_1) and employing the above procedure, it is seen that the triple (a,c_1,c_2) where

$$c_2=9\alpha+i\beta(18k+12)$$

Exhibits diophantine 3-tuple with property $D(-\beta^2)$

Taking (a,c_2) and employing the above procedure, it is seen that the triple (a,c_2,c_3) where

$$c_3 = 16\alpha + i\beta(32k + 24)$$

Exhibits diophantine 3-tuple with property $D(-\beta^2)$

Taking (a, c_3) and employing the above procedure, it is seen that the triple (a, c_3, c_4) where

$$c_4 = 25\alpha + i\beta(50k + 40)$$

Exhibits diophantine 3-tuple with property $D(-\beta^2)$

The repetition of the above process leads to the generation of sequence of diophantine 3-tuples whose general form is given by $(a, c_{\partial-1}, c_{\partial})$ where

$$c_{\partial-1} = \partial^2\alpha + i\beta\left((2k+2)\partial^2 - 2\partial\right),\ \partial = 1, 2, 3, ...$$

Table 14: Numerical Examples with property $D(-\beta^2)$

k	α	β	**(a, c₀, c₁)**	**(a₁,c₁,c₂)**	**(a,c₂, c₃)**
1	3	2	(3+i8, 3+i4, 12+i24)	(3 +i8, 12+i24, 27+i60)	(3+i8, 27+i60, 48+ill2)
2	4	3	(4+i18, 4+i12, 16+i60)	(4+i18, 16+i60, 36+i144)	(4+i18, 36+i144, 64+i264)
3	5	4	(5+i32, 5+i24, 20+i112)	(5+i32, 20+i112, 45+i264)	(5+i32, 45+i264, 80+i480)
4	6	5	(6+i50, 6+i40, 24+i180)	(6+i50, 24+i180, 54+i420)	(6+i50, 54+i420, 96+i760)
5	7	6	(7+i72, 7+i60, 28+i264)	(7+i72, 28+i264, 63+i612)	(7+i72, 63+i612, 112+i1104)

Sequence: 15

Let $a = \alpha + i(2k+1)\beta,\ \ c_0 = \alpha + i(2k-1)\beta$

It is observed that

$$ac_0 - \beta^2 = (\alpha + i2k\beta)^2$$

Therefore, the pair (a, c_0) represents diophantine 2-tuple with the property $D(-\beta^2)$.

Let c_1 be any non-zero polynomial such that

$$ac_1 - \beta^2 = p^2 \quad (57)$$

$$c_0c_1 - \beta^2 = q^2 \quad (58)$$

Eliminating c_1 between (57) and (58), we have

$$c_0p^2 - aq^2 = (a - c_0)\beta^2 \quad (59)$$

Introducing the linear transformations

$$p = X + aT \ , \ q = X + c_0 T \tag{60}$$

In (59) and simplifying we get

$$X^2 = ac_0 T^2 - \beta^2$$

Which is satisfied by $T = 1 \ , \ X = \alpha + i2k\beta$

In view of (60) and (57), it is seen that

$$c_1 = 4\alpha + i8k\beta$$

Note that (a, c_0, c_1) represents diophantine 3-tuple with property $D(-\beta^2)$

Taking (a, c_1) and employing the above procedure, it is seen that the triple (a, c_1, c_2) where

$$c_2 = 9\alpha + i\beta(18k + 3)$$

Exhibits diophantine 3-tuple with property $D(-\beta^2)$

Taking (a, c_2) and employing the above procedure, it is seen that the triple (a, c_2, c_3) where

$$c_3 = 16\alpha + i\beta(32k + 8)$$

Exhibits diophantine 3-tuple with property $D(-\beta^2)$

Taking (a, c_3) and employing the above procedure, it is seen that the triple (a, c_3, c_4) where

$$c_4 = 25\alpha + i\beta(50k + 15)$$

Exhibits diophantine 3-tuple with property $D(-\beta^2)$

The repetition of the above process leads to the generation of sequence of diophantine 3-tuples whose general form is given by $(a, c_{\partial-1}, c_{\partial})$where

$$c_{\partial-1} = \partial^2\alpha + i\beta\ ((2k+1)\partial^2 - 2\partial),\ \partial = 1, 2, 3, ...$$

Table 15: Numerical Examples with property $D(-\beta^2)$

k	α	β	**(a,c_0,c_1)**	**(a, c_1, c_2,)**	**(a,c_2,c_3)**
1	3	2	(3+i6, 3+i2, 12+i16)	(3+i6, 12+i16, 27+i42)	(3+i6, 27+i42, 48+i80)
2	4	3	(4+i15, 4+i9, 16+i48)	(4+i15, 16+i48, 36+ill7)	(4+i15, 36+i117,64+i216)
3	5	4	(5+i28, 5+i20, 20+i96)	(5+i28, 20+i96, 45+i228)	(5+i28, 45+i228, 80+i416)
4	6	5	(6+i45, 6+i35, 24+i160)	(6+i45, 24+i160, 54+i375)	(6+i45, 54+i375, 96+i680)
5	7	6	(7+i66, 7+i54, 28+i240)	(7+i66, 28+i240, 63+i558)	(7+i66, 63+i558, 112+i1008)

Sequence: 16

Let $a = \alpha + i(2k-1)\beta, \quad c_0 = \alpha + i(2k+1)\beta$

It is observed that

$$ac_0 - \beta^2 = (\alpha + i2k\beta)^2$$

Therefore, the pair (a, c_0) represents diophantine 2-tuple with the property $D(-\beta^2)$.

Let c_1 be any non-zero polynomial such that

$$ac_1 - \beta^2 = p^2 \tag{61}$$

$$c_0 c_1 - \beta^2 = q^2 \tag{62}$$

Eliminating c_1 between (61) and (62), we have

$$c_0 p^2 - aq^2 = (a - c_0)\beta^2 \tag{63}$$

Introducing the linear transformations

$$p = X + aT \ , \ q = X + c_0 T \tag{64}$$

In (63) and simplifying we get

$$X^2 = ac_0 T^2 - \beta^2$$

Which is satisfied by $T = 1$, $X = \alpha + i2k\beta$

In view of (64) and (61), it is seen that

$$c_1 = 4\alpha + i8k\beta$$

Note that (a, c_0, c_1) represents diophantine 3-tuple with property $D(-\beta^2)$

Taking (a, c_1) and employing the above procedure, it is seen that the triple (a, c_1, c_2) where

$$c_2 = 9\alpha + i\beta(18k - 3)$$

Exhibits diophantine 3-tuple with property $D(-\beta^2)$

Taking (a, c_2) and employing the above procedure, it is seen that the triple (a, c_2, c_3) where

$$c_3 = 16\alpha + i\beta(32k - 8)$$

Exhibits diophantine 3-tuple with property $D(-\beta^2)$

Taking (a, c_3) and employing the above procedure, it is seen that the triple (a, c_3, c_4) where

$$c_4 = 25\alpha + i\beta(50k - 15)$$

Exhibits diophantine 3-tuple with property $D(-\beta^2)$

The repetition of the above process leads to the generation of sequence of diophantine 3-tuples whose general form is given by $(a, c_{\partial-1}, c_{\partial})$ where

$$c_{\partial-1} = \partial^2\alpha + i\beta\left((2k-1)\partial^2 + 2\partial\right),\ \partial = 1, 2, 3, ...$$

Table 16: Numerical Examples with property $D(-\beta^2)$

k	α	β	(a, c_0, c_1)	(a, c_1, c_2)	(a, c_2, c_3)
1	3	2	(3+i2, 3+i6, 12+i16)	(3+i2, 12+i16, 27+i30)	(3+i2, 27+i30, 48+i48)
2	4	3	(4+i9, 4+i15, 16+i48)	(4+i9, 16+i48, 36+i99)	(4+i9, 36+i99, 64+i168)
3	5	4	(5+i20, 5+i28, 20+i96)	(5+i20, 20+i96, 45+i204)	(5+i20, 45+i204, 80+i352)
4	6	5	(6+i35, 6+i45, 24+i160)	(6+i35, 24+i160, 54+i345)	(6+i35, 54+i345, 96+i600)
5	7	6	(7+i54, 7+i66, 28+i240)	(7+i54, 28+i240, 63+i522)	(7+i54, 63+i522, 112+i912)

Sequence: 17

Let $a = \alpha + i\beta - 2s,\ \ c_0 = \alpha + i\beta + 2s$

It is observed that

$$ac_0 + 4s^2 = (\alpha + i\beta)^2$$

Therefore, the pair (a, c_0) represents diophantine 2-tuple with the property $D(4s^2)$.

Let c_1 be any non-zero polynomial such that

$$ac_1 + 4s^2 = p^2 \tag{65}$$

$$c_0 c_1 + 4s^2 = q^2 \tag{66}$$

Eliminating c_1 between (65) and (66), we have

$$c_0 p^2 - aq^2 = (c_0 - a)(4s^2) \tag{67}$$

Introducing the linear transformations

$$p = X + aT \ , \ q = X + c_0 T \tag{68}$$

In (67) and simplifying we get

$$X^2 = ac_0 T^2 + 4s^2$$

Which is satisfied by $T = 1 \ , \ X = \alpha + i\beta$

In view of (68) and (65), it is seen that

$$c_1 = 4(\alpha + i\beta)$$

Note that (a, c_0, c_1) represents diophantine 3-tuple with property $D(4s^2)$

Taking (a, c_1) and employing the above procedure, it is seen that the triple (a, c_1, c_2) where

$$c_2 = 9(\alpha + i\beta) - 6s$$

Exhibits diophantine 3-tuple with property $D(4s^2)$

Taking (a, c_2) and employing the above procedure, it is seen that the triple (a, c_2, c_3) where

$$c_3 = 16(\alpha + i\beta) - 16s$$

Exhibits diophantine 3-tuple with property $D(4s^2)$

Taking (a, c_3) and employing the above procedure, it is seen that the triple (a, c_3, c_4) where

$$c_4 = 25(\alpha + i\beta) - 30s$$

Exhibits diophantine 3-tuple with property $D(4s^2)$

The repetition of the above process leads to the generation of sequence of diophantine 3-tuples whose general form is given by $(a, c_{\partial-1}, c_{\partial})$ where

$$c_{\partial-1} = \partial^2(\alpha + i\beta) + (-2\partial^2 + 4\partial)s, \ \partial = 1, 2, 3, ...$$

Table 17: Numerical Examples with property $D(4s^2)$

k	α	β	s	(a,c_0,c_1)	(a, c_1,c_2)	(a,c_2,c_3)
1	3	2	4	(-5+i2,11+i2,12+i8)	(-5+i2,12+i8,3+i18)	(-5+i2,3+i18,-16+i32)
2	4	3	5	(-6+i3,14+i3, 16+i12)	(-6+i3,16+i12,6+i27)	(-6+i3,6+i27,-16+i48)
3	5	4	6	(-7+i4,17+i4,20+i16)	(-7+i4,20+i16,9+i36)	(-7+i4,9+i36,-16+i64)
4	6	5	7	(-8+i5,20+i5,24+i20)	(-8+i5,24+i20,12+i45)	(-8+i5,12+i45,-16+i80)
5	7	6	8	(-9+i6,23+i6,28+i24)	(-9+i6, 28+i24,15+i54)	(-9+i6,15+i54, -16+i96)

Sequence: 18

Let $a = \alpha + i\beta + 2s, \ c_0 = \alpha + i\beta - 2s$

It is observed that

$$ac_0 + 4s^2 = (\alpha + i\beta)^2$$

Therefore, the pair (a, c_0) represents diophantine 2-tuple with the property $D(4s^2)$.

Let c_1 be any non-zero polynomial such that

$$ac_1 + 4s^2 = p^2 \tag{69}$$

$$c_0c_1 + 4s^2 = q^2 \tag{70}$$

Eliminating c_1 between (69) and (70), we have

$$c_0p^2 - aq^2 = (c_0 - a)(4s^2) \tag{71}$$

Introducing the linear transformations

$$p = X + aT \ , \ q = X + c_0T \tag{72}$$

In (71) and simplifying we get

$$X^2 = ac_0 T^2 + 4s^2$$

Which is satisfied by $T = 1 \ , X = \alpha + i\beta$

In view of (80) and (69), it is seen that

$$c_1 = 4(\alpha + i\beta)$$

Note that (a, c_0, c_1) represents diophantine 3-tuple with property $D(4s^2)$

Taking (a, c_1) and employing the above procedure, it is seen that the triple (a, c_1, c_2) where

$$c_2 = 9(\alpha + i\beta) + 6s$$

Exhibits diophantine 3-tuple with property $D(4s^2)$

Taking (a, c_2) and employing the above procedure, it is seen that the triple (a, c_2, c_3) where

$$c_3 = 16(\alpha + i\beta) + 16s$$

Exhibits diophantine 3-tuple with property $D(4s^2)$

Taking (a, c_3) and employing the above procedure, it is seen that the triple (a, c_3, c_4) where

$$c_4 = 25(\alpha + i\beta) + 30s$$

Exhibits diophantine 3-tuple with property $D(4s^2)$

The repetition of the above process leads to the generation of sequence of diophantine 3-tuples whose general form is given by $(a, c_{\partial-1}, c_{\partial})$ where

$$c_{\partial-1} = \partial^2(\alpha + i\beta) + (2\partial^2 - 4\partial)s, \ \partial = 1, 2, 3, ...$$

Table 18: Numerical Examples with property $D(4s^2)$

k	α	β	s	**(a,c₀,c₁)**	**(a, c₁, c₂)**	**(a,c₂,c₃)**
1	3	2	4	(11+i2, -5+i2, 12+i8)	(11+i2, 12+i8, 51+i18)	(11+i2, 51+i18, 112+i32)
2	4	3	5	(14+i3, -6+i3, 16 +i12)	(14+i3, 16+i12, 66+i27)	(14+i3, 66+i27, 144+i48)
3	5	4	6	(17+i4, -7+i4, 20+i16)	(17+i4, 20+i16, 81+i36)	(17+i4, 81+i36, 176+i64)
4	6	5	7	(20+i5, -8+i5, 24+i20)	(20+i5, 24+i20, 96+i45)	(20+i5, 96+i45, 208+i180)
5	7	6	8	(23+i6, -9+i6, 28+i24)	(23+i6, 28+i24, 111+i54)	(23+i6, 111+i54, 240+i96)

References

1. I.G. Bashmakova, ed., Diophantus of Alexandria, Arithmetics and the Book of Polygonal Numbers, Nauka, Moscow, 1974.
2. A.F. Beardon and M.N. Deshpande, Diophantine triples, Math. Gazette 86, 258-260, 2002.
3. Carmicheal RD. The Theory of Numbers and Diophantine Analysis, Dover Publications, New York, 1959.
4. Dujella. The problem of the extension of a parametric family of Diophantine triples, Publ. Math. Breen. 1997; 51:311-322.
5. Gopalan MA, Srividhya G, Diophantine Quadruple for Fibonacci and Lucas Numbers with property D(4), Diophantus Journal of Mathematics. 2012; 1(1):15-18.
6. Gopalan MA, Srividhya G. Some non-extendable P-5 sets, Diophantus Journal of Mathematics. 2012; 1(1):19-22.
7. Deshpande MN. One interesting family of Diophantine triples. Internet J. Math. ed. Sci. Tech. 2012; 33:253-256.
8. M.A. Gopalan and V. Geetha, Sequences of Diophantine triples, JP Journal of Mathematical Sciences, Volume 14, Issues 1 & 2, 27-39, 2015.
9. M.A. Gopalan and V. Geetha, Formation of Diophantine Triples for Polygonal Numbers $\left(t_{16,n} \text{ to } t_{25,n}\right)$ and Centered Polygonal Numbers $\left(ct_{16,n} \text{ to } ct_{25,n}\right)$, IJITR, volume 3, Issue 1, 1837-1841, December-January 2015.
10. G. Janaki and S. Vidhya, Construction of the diophantine triple involving Stella octangula number, Journal of Mathematics and Informatics, vol.10, Special issue, 89-93, December 2017.
11. S. Vidhyalakshmi, J. Shanthi, T. Mahalakshmi, M. A. Gopalan, On Curious Family of 3-Tuples, Publish Book Online, Page N0: 1-56.
12. G. Janaki and S. Vidhya, Construction of the Diophantine Triple involving Pronic Number, IJRASET, Volume 6, Issue I, 2201-2204,

January 2018.

13. G. Janaki and C. Saranya, Construction of the Diophantine Triple involving Pentatope Number, IJRASET, Volume 6, Issue III, 2317-2319, March 2018.
14. J.Shanthi, M.A.Gopalan and Sharadha Kumar, On Sequences of Diophantine 3-Tuples through Euler Polynomials, IJAST, Vol 27, No.1, (2019), Pp:318-325.
15. A.Vijayasankar, Sharadha Kumar, M.A.Gopalan, On Sequences of diophantine 3-tuples generated through Pronic Numbers, IOSR-JM, 15(5) ser.II, (Sep-Oct 2019), Pp:41-46.
16. M.A.Gopalan, Sharadha Kumar, On Sequences of Diophantine 3-tuples generated through Euler and Bernoulli Polynomials, Tamap Journal of Mathematics and Statistics, Volume 2019,(2019), Pp:1-5.